

Introduction

High precision oscillators often require measuring frequency differences of a few parts in 10^{12} , or less. These tests are usually made when adjusting precision oscillators to a known frequency standard or when studying aging effects. With present methods of precision frequency comparisons, the greater the resolution required, the longer it takes to make the measurement.¹ As an example, it would take about a day to compare two 5 MHz frequency standards to 1 part in 10^{13} . By measuring phase shift versus time with the 8405A Vector Voltmeter, 1 part in 10^{13} frequency difference can be measured in a few minutes.

The 8405A Vector Voltmeter is a two channel RF millivoltmeter and phasemeter. It compares the phase of two RF signals with very high precision. The technique results in frequency comparisons to $1/10^{13}$ within a few minutes time at typical standard frequencies of 1 MHz and above. Input requirements to the 8405A are that the initial frequency difference between signals be less than 1 or 2 Hz and that both signals are in the 8405A range of 1 MHz to 1 GHz. Oscillators which are initially off frequency by more than 1 or 2 Hz should first be rough tuned using normal indicators such as a counter or oscilloscope². The signals must also have low noise. These requirements are typically met by precision oscillators such as the Hewlett-Packard 101A, 106A/B, 107AR/BR, and 5060A or equivalent.

Adjustments previously requiring days and even weeks to accomplish are now made in minutes using the 8405A. Short term stability and fractional frequency deviation studies are practical on the most precise oscillator. A continuous record of frequency difference is easily obtained by connecting a strip chart recorder to the 8405A.

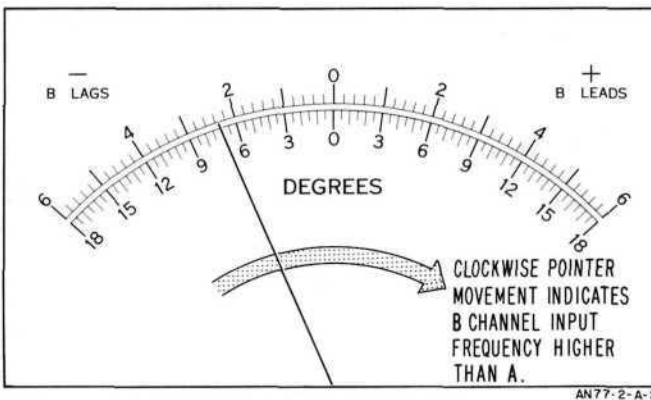
¹HP Application Note 52, "Frequency and Time Standards," pg.3-1.

²Ibid, pg.3-2.

Description

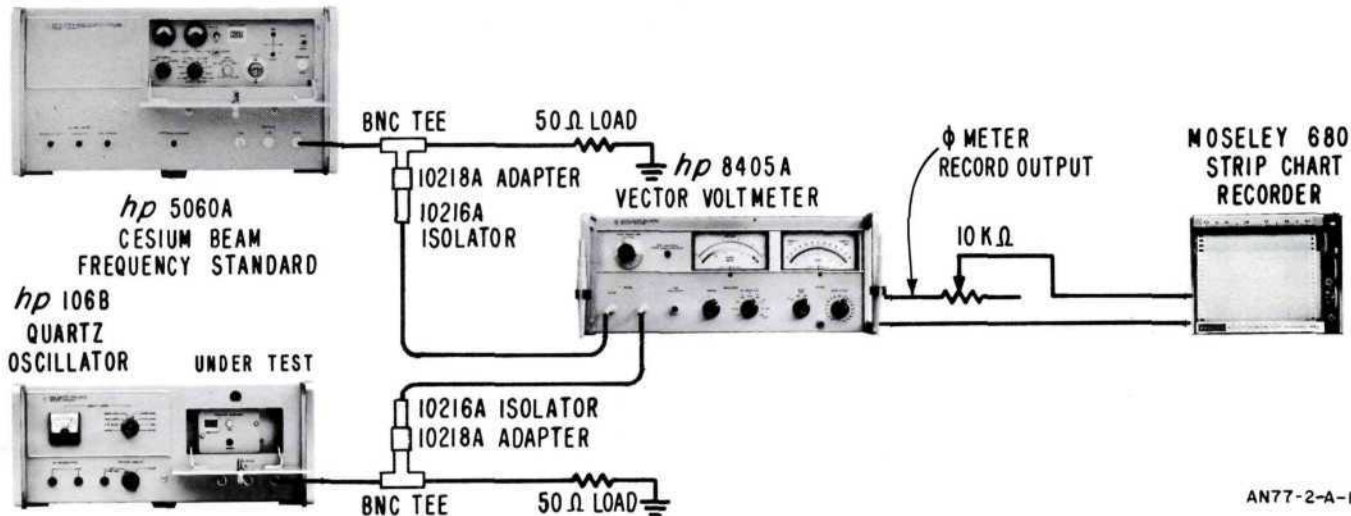
Application of the 8405A to precise frequency comparisons is very simple and straightforward. The Channel A and B inputs of the instruments are connected to the outputs of the two frequency sources to be compared, as shown in Figure 1.

The phase of Channel B is compared to Channel A and indicated on the phasemeter. A DC recorder jack on the rear panel of the 8405A provides a voltage output proportional to the phasemeter reading. (Polarity indicates lead or lag of B with respect to A.) Any frequency difference between A and B inputs causes a continuous change in phase reading. If the phasemeter indicates a clockwise change, as in Figure 2, then Channel B input frequency is higher than A. Counterclockwise rotation, of course, indicates that the B input is the lower frequency. The phase change and direction of change can be recorded over a known period of time using a strip chart recorder. Frequency difference, in proportional parts, can then be calculated and read from a calibrated recorder.



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Figure 2. Phasemeter of 8405A indicates frequency of B channel higher than A channel when indicator rotates clockwise.



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Figure 1. Cesium Beam and Quartz Oscillator frequencies are measured with high resolution when connected to the 8405A Vector Voltmeter as shown above.

Calculation

To show how phase change relates to frequency difference, consider an input signal of 5 MHz to Channel A and 5 MHz + 1 Hz to Channel B. The phase angle between A and B will change by 360° every second for every cycle of frequency difference between A and B. This phase shift is read directly from the 8405A phase meter. By measuring the time required for the phase change, the frequency difference can be calculated.

Since $360^\circ/\text{sec} = 1 \text{ Hz (cps)}$, then

$$\frac{360^\circ/\text{sec}}{1 \text{ Hz}} = \frac{\Delta\phi/\Delta t}{\Delta f}$$

Solving for Δf ,

$$(1) \quad \Delta f = \frac{\Delta\phi}{360^\circ\Delta t}$$

Δt = time, in seconds, required for phase change measured above

Δf = frequency difference between input signals

$\Delta\phi$ = phase change in degrees (indicated on 8405A)

Dividing Δf by the frequency of the reference oscillator then gives the frequency difference in proportional parts.

Example:

Standard frequency $f = 5 \text{ MHz}$

$\Delta\phi$ measured on 8405A = 1.3°

$\Delta t = 60 \text{ seconds}$

$$\begin{aligned} \frac{\Delta f}{f} &= \frac{1.3^\circ}{360^\circ (60 \text{ sec}) (5 \times 10^6 \text{ Hz})} \\ &= \frac{1.3}{1.08 \times 10^{11}} = 1.2 \times 10^{-11} \end{aligned}$$

or $1.2 \text{ parts}/10^{11}$

In this example the 1.3° phase change was clockwise indicating Channel B input was leading Channel A. Therefore, Channel B input frequency was higher than A by $1.2 \text{ parts in } 10^{11}$.

In general, the frequency difference between two oscillators is determined from the slope of phase change versus time.

$$\begin{aligned} \omega &= \frac{d\phi}{dt} & f &= \text{frequency difference between two sources} \\ f &= \frac{1}{2\pi} \left(\frac{d\phi}{dt} \right) & \omega &= 2\pi f \\ f &= \frac{1}{360^\circ} \left(\frac{d\phi}{dt} \right) & \phi &= \text{phase difference between two sources} \\ & & \frac{d\phi}{dt} &= \text{instantaneous change of phase, or slope of phase versus time (deg/sec)} \end{aligned}$$

Therefore, if the trace is nonlinear, the phase slope at any time will be proportional to the instantaneous frequency difference between the two sources. A constant frequency offset appears as a linear phase versus time trace. A drift in frequency results in a curved or nonlinear trace.

Resolution

Frequency comparison is required for two general applications: (1) when adjusting an oscillator to agree with a standard, and (2) for continuous monitoring of oscillators during aging studies or changing environmental conditions. The resolution required for adjusting an oscillator is usually greater than for long term monitoring applications. In order to obtain higher resolution, we can expand the phase meter sensitivity to $\pm 6^\circ$ full scale instead of $\pm 180^\circ$. On the 6° range, resolution is 0.1° . From Equation (1) a phase change of $.1^\circ$ in one second corresponds to $2.777 \times 10^{-4} \text{ Hz}$ frequency difference. At 5 MHz this represents $5.555 \text{ parts}/10^{11}$.

By monitoring phase change over periods greater than 1 second, we obtain even greater resolution. This is the usual technique and it is easily made by using a strip chart recorder on the phase meter output. For example, a phase change of 0.1° in 10 seconds increases the resolution of measurement to $5.555 \text{ parts}/10^{12}$. A 0.1° change in 100 seconds is $5.555 \text{ parts}/10^{13}$, and so on.

From these examples, we can see that in less than 2 minutes of monitoring time with the 8405A, we are able to compare and adjust frequency standards and highly stable oscillators with very high precision.

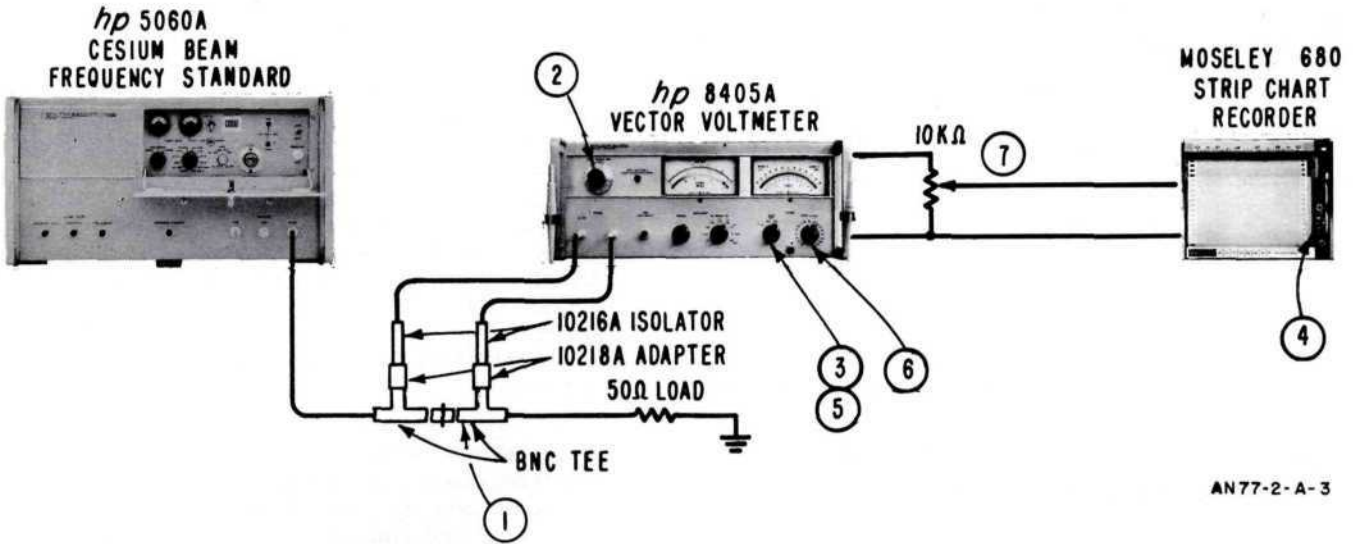
Measurement

The phase meter offset and range controls are used to calibrate recorder span directly in degrees. Figure 3 shows the calibration method for full scale readings of $\pm 5^\circ$, 15° , 50° , or 150° , depending on the phase range selected. These ranges correspond with the 5 inch chart of the Moseley 680 recorder. Expanded ranges of 1° , 3° , 10° , and 30° full scale can be set up using the procedure shown in Figure 4.

High Resolution Measurements

With a recorder range sensitivity of 1 volt, a 0.1° change in phase corresponds to only a very small deflection on the recorder output. (See Figure 5a.) It is, therefore, desirable to make the recorder span more sensitive than 1 volt in order to properly resolve $.1^\circ$ or smaller phase changes. To do this, the recorder range is set for 50 millivolts. Figure 5(b) shows the increased resolution. This method is useful for making fine adjustments to an oscillator and gives maximum resolution when needed. For long term monitoring, the recorder span is reduced and/or the 8405A phase range increased as required to keep the recorder on scale over long periods. A calibration and test procedure for higher resolution are given in Figure 4.

CALIBRATE



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1. Connect both probes to frequency standard using accessory tees.
2. Set 8405A **FREQ RANGE** to agree with input frequency.
3. Zero phase meter on $\pm 6^\circ$ range using **PHASE METER ZERO**.
4. Set recorder range to 1 volt. Set pen to center scale with recorder zero control.
5. Set phase range to $\pm 60^\circ$.
6. Change phase meter offset by $+50^\circ$.
7. Adjust 10K ohm pot for full scale pen deflection on recorder.

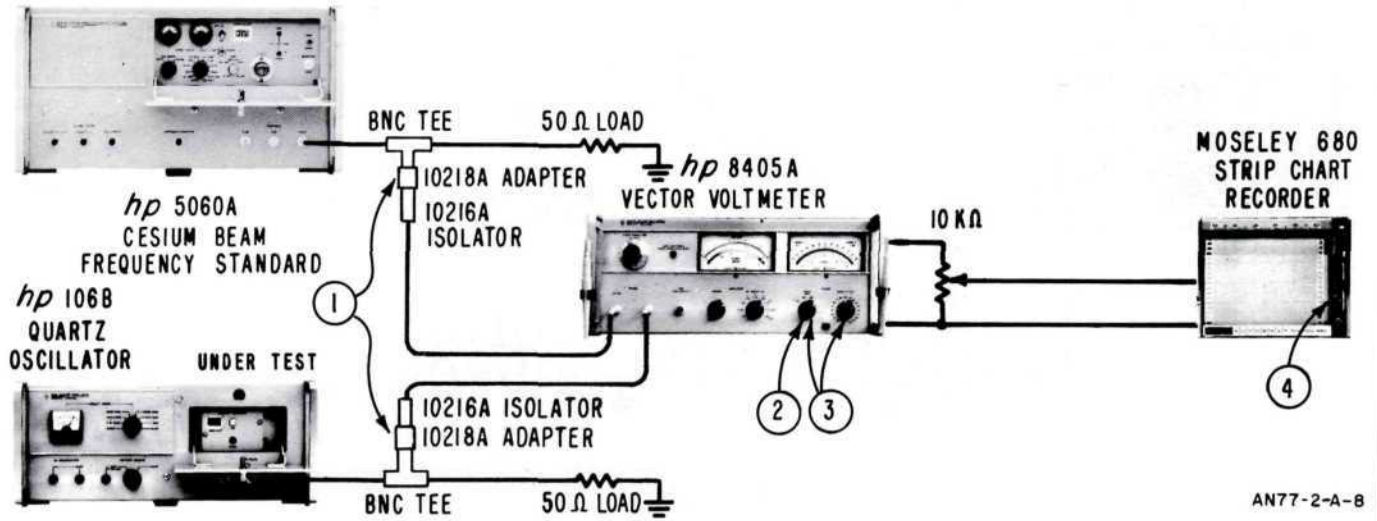
8. Change phase meter offset polarity to minus and check pen deflection on the opposite side of zero. Make required fine adjustments to recorder zero and 10K pot for equal deflection from center scale.

THE RECORDER IS NOW CALIBRATED FOR EACH PHASE RANGE SELECTED ON THE VECTOR VOLTMEETER AS FOLLOWS:

8405A PHASE RANGE	680 RECORDER CALIBRATION
$\pm 6^\circ$	0.2°/minor div. ($\pm 5^\circ$ f. s.)
$\pm 18^\circ$	0.6°/ " " ($\pm 15^\circ$ f. s.)
$\pm 60^\circ$	2°/ " " ($\pm 50^\circ$ f. s.)
$\pm 180^\circ$	6°/ " " ($\pm 150^\circ$ f. s.)

Figure 3. Measurement procedure for comparisons of frequency standards (sheet 1 of 2).

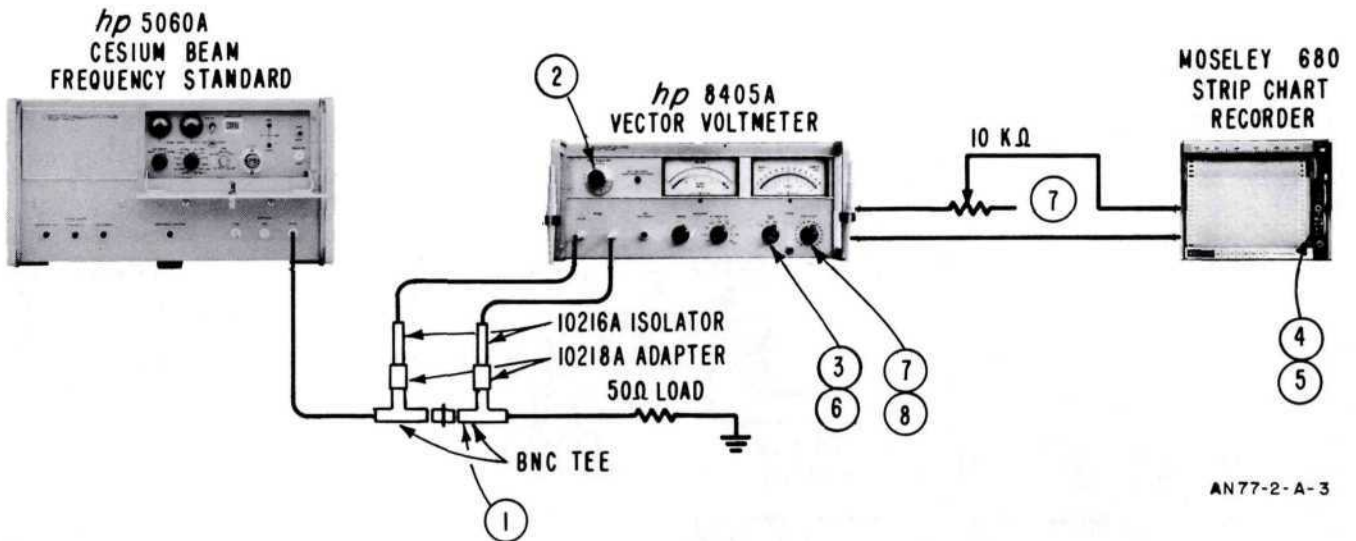
TEST



1. Connect meter probes to frequency standard and device in test.
2. Set PHASE RANGE for desired resolution.
3. Set PHASE METER ZERO and OFFSET for zero degrees indicated on the recorder and start the strip chart drive at desired speed.
4. Record phase change over desired time.
5. Determine frequency difference between A and B inputs using Equation (1) or the charts in Appendix I.

Figure 3. Measurement procedure for comparisons of frequency standards (sheet 2 of 2).

CALIBRATE



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1. Connect both probes to frequency standard using accessory tees.
2. Set 8405A **FREQ RANGE** to agree with input frequency.
3. Zero phase meter on $\pm 6^\circ$ range using **PHASE METER ZERO**.
4. Set recorder input **RANGE** to 50 mV.

Note

Input resistance to Moseley 680 recorder is 10K ohms on the 50 mV range (200K ohms/volt). Rheostat connection to 10K ohm variable resistor provides variable span adjust without loading 8405A output.

5. Set recorder pen to left hand edge of chart with recorder **ZERO**.
6. Set **PHASE RANGE** to $\pm 60^\circ$.
7. Change phase meter **OFFSET** by 10° and adjust

10K ohm rheostat for full scale pen deflection. (If pen drives hard left, change phase meter offset polarity switch setting.)

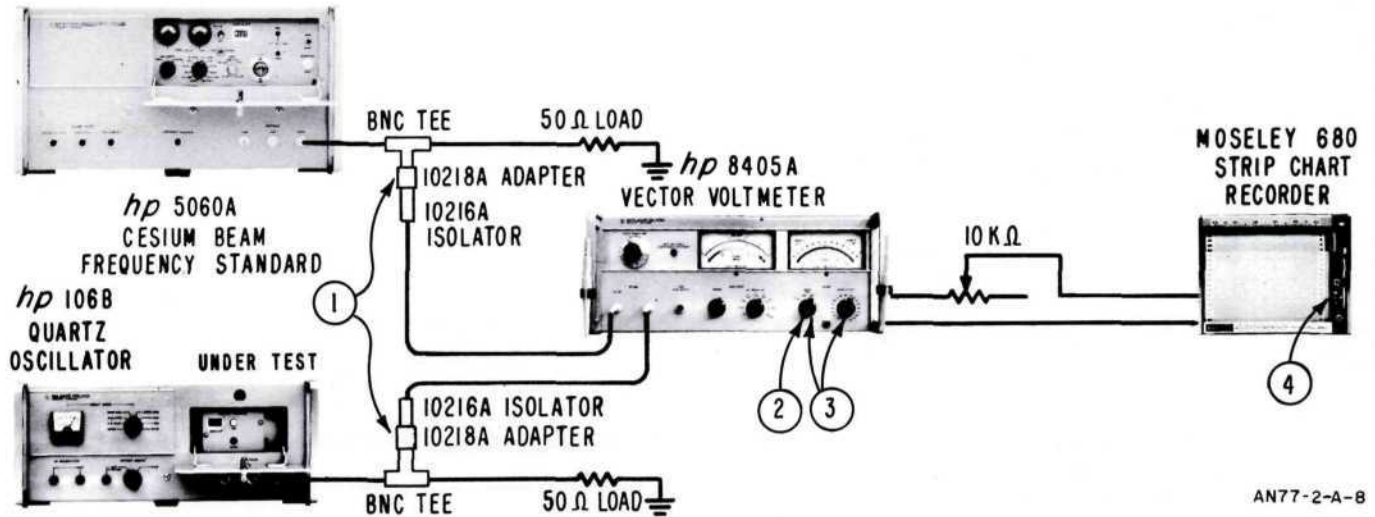
8. Return to zero offset and check recorder zero adjustment.

THE RECORDER IS NOW CALIBRATED FOR EACH PHASE RANGE SELECTED ON THE VECTOR VOLTMEETER AS FOLLOWS:

8405A <u>PHASE RANGE</u>	680 <u>RECORDER CALIBRATION</u>
$\pm 6^\circ$.02°/minor div. (1°f. s.)
$\pm 18^\circ$.06°/ " " (3°f. s.)
$\pm 60^\circ$	0.2°/ " " (10°f. s.)
$\pm 180^\circ$	0.6°/ " " (30°f. s.)

Check 8405A stability by running the strip chart recorder for a given period of time. This check will verify that the stability is sufficient to make high resolution measurements.

Figure 4. Measurement procedure for high resolution comparisons of frequency standards (sheet 1 of 2).

TEST

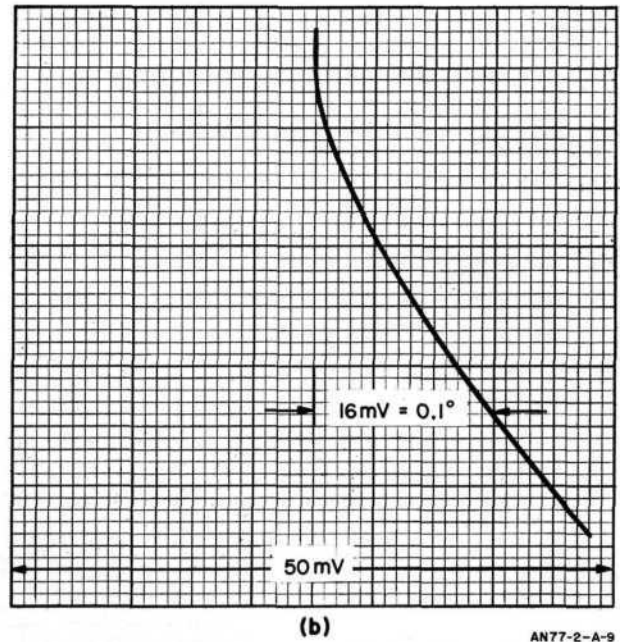
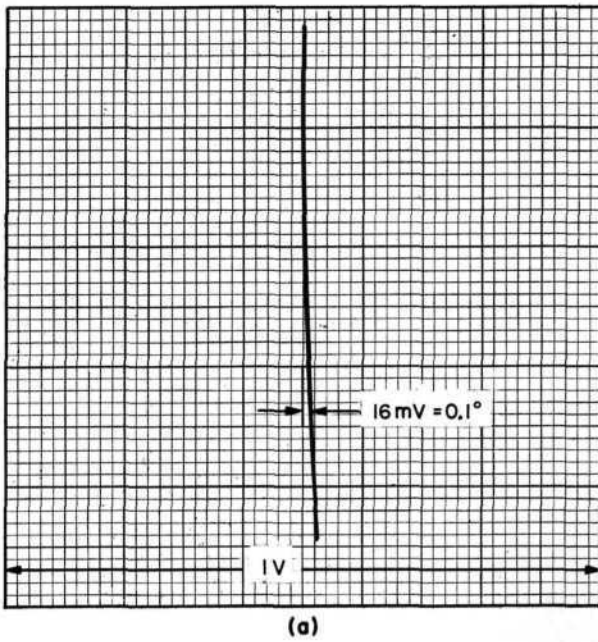
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1. Connect meter probes for desired test.
2. Set PHASE RANGE for desired resolution.
3. Set phase meter ZERO and OFFSET for desired pen deflection on the recorder. (A center chart setting will make the meter indicate off zero.

This does not cause any error because the measurement is phase change over a period of time. Any starting point may be chosen.)

4. Start the recorder chart drive at desired speed and record phase change for desired time.
5. Determine frequency difference between A and B inputs using Equation (1) or the charts in Appendix I.

Figure 4. Measurement procedure for high resolution comparisons of frequency standards (sheet 2 of 2).



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Figure 5. Strip chart recordings show resolution of 0.1° phase change on a recorder span of:
 a) 1 volt full scale
 b) 50 mV full scale

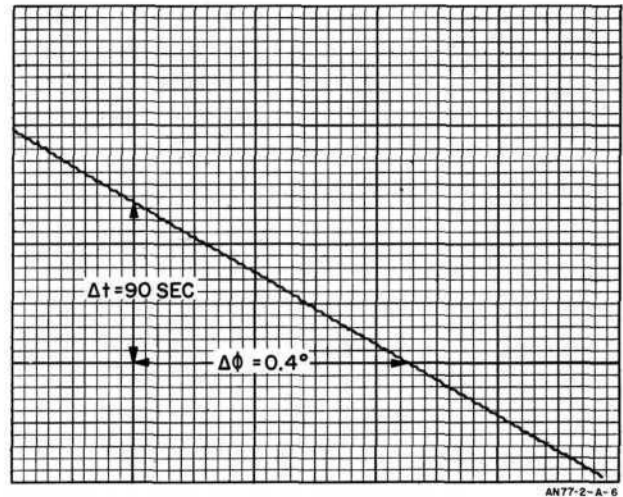
Graphical Solution

The charts in Appendix I solve Equation (1) for standard frequencies of 1 MHz and 5 MHz respectively. Proportional frequency difference is found by entering the chart at $\Delta\phi$ and Δt values indicated on the calibrated recorder. The point where $\Delta\phi$ and Δt intersect is read from the radial line calibrations directly in proportional parts.

All scales of the chart have decade relationships. Resolution is possible for phase changes of tenths and hundredths of a degree by using the appropriate scale factor.

Example:

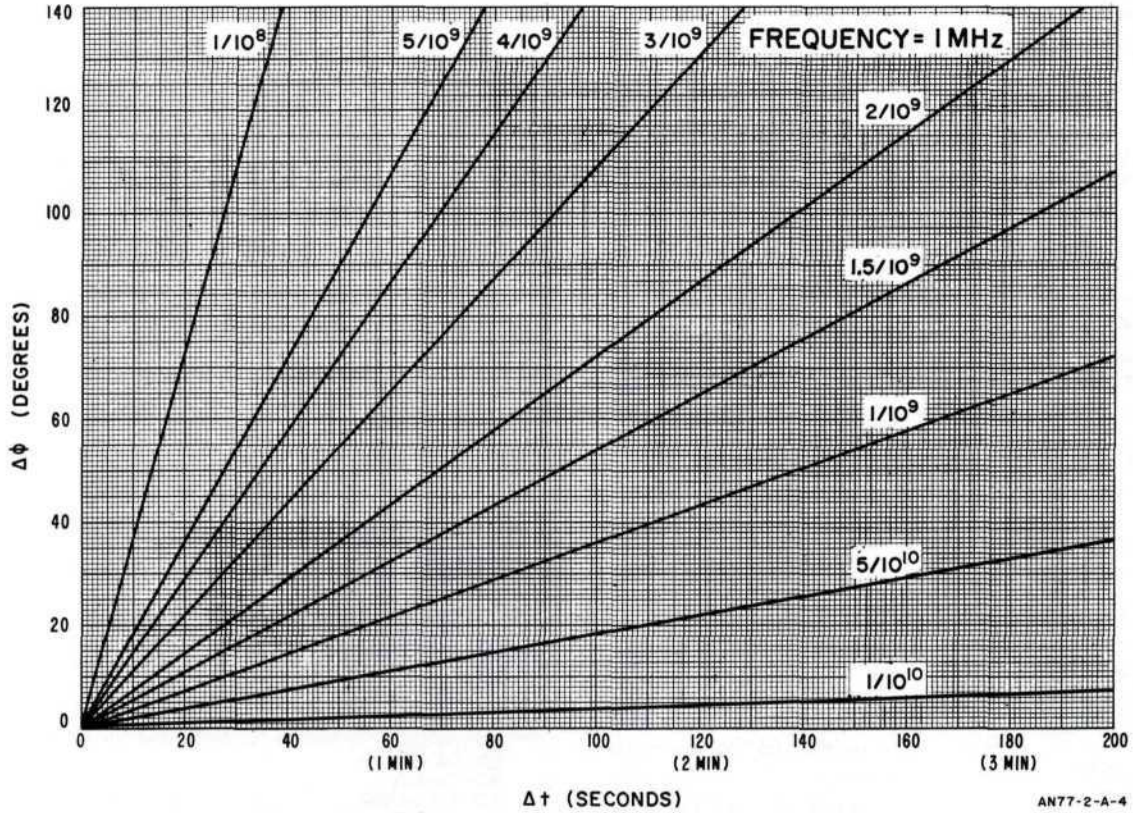
Figure 6 is a strip chart record of phase change between two hp 107BR Quartz Oscillators at 1 MHz. Note that phase change is only 0.4° in 90 seconds time. Enter the conversion chart in Appendix I at 40° , instead of 0.4° . Because of the 100:1 change in $\Delta\phi$ scale, all radial line calibrations change by 100:1. For example, the $1/10^9$ line becomes $1/10^{11}$. Now intersect the Δt scale at 90 seconds and find the frequency difference between the oscillators as $1.25/10^{11}$.



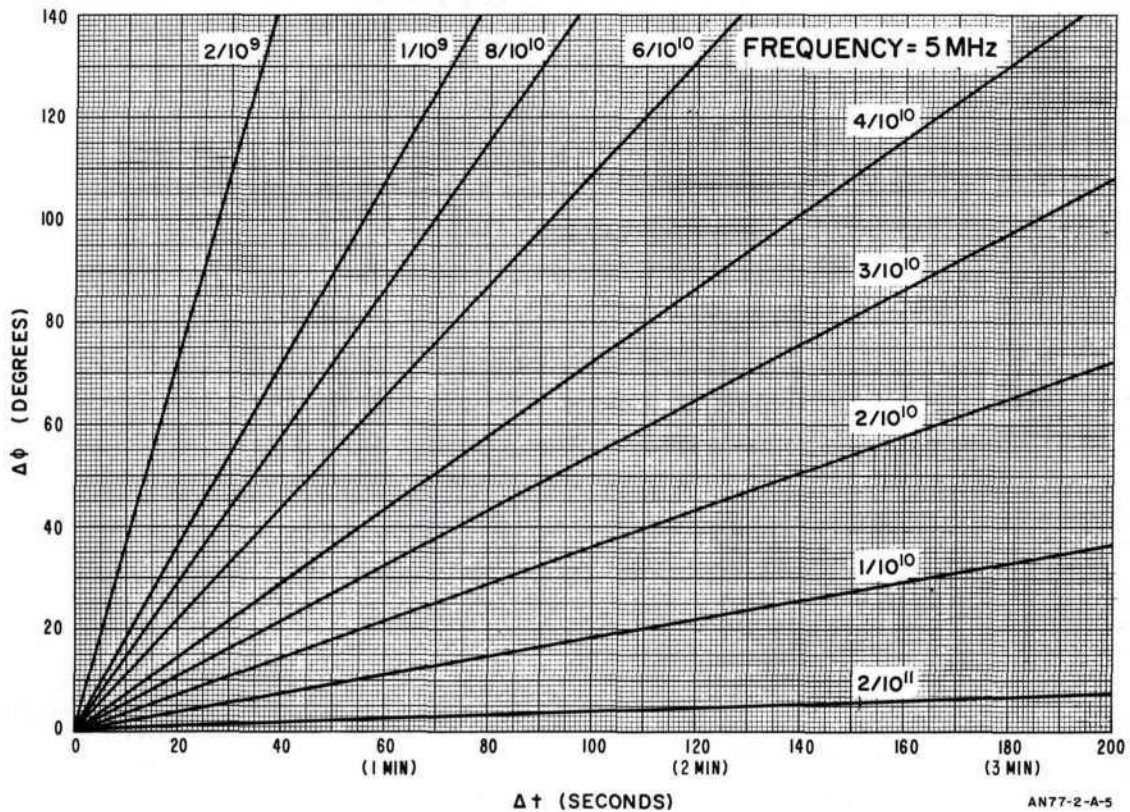
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Figure 6. Typical Strip Chart Recording of phase between 2 hp 1 MHz 107BR Quartz Oscillators. Phase change of 0.4° in 90 seconds indicates frequency offset is 1.25 parts in 10^{11} .

APPENDIX I



Graphical solution of $\frac{\Delta f}{f} = \frac{1}{360^\circ} \left(\frac{\Delta \phi}{\Delta t} \right)$ for frequency difference between two 1 MHz oscillators.



Same as above, except frequency of standards is 5 MHz.