Application Note 358-2

HEWLETT PACKARD

HP 5371A Frequency and Time Interval Analyzer
Jitter and Wander Analysis in Digital Communications
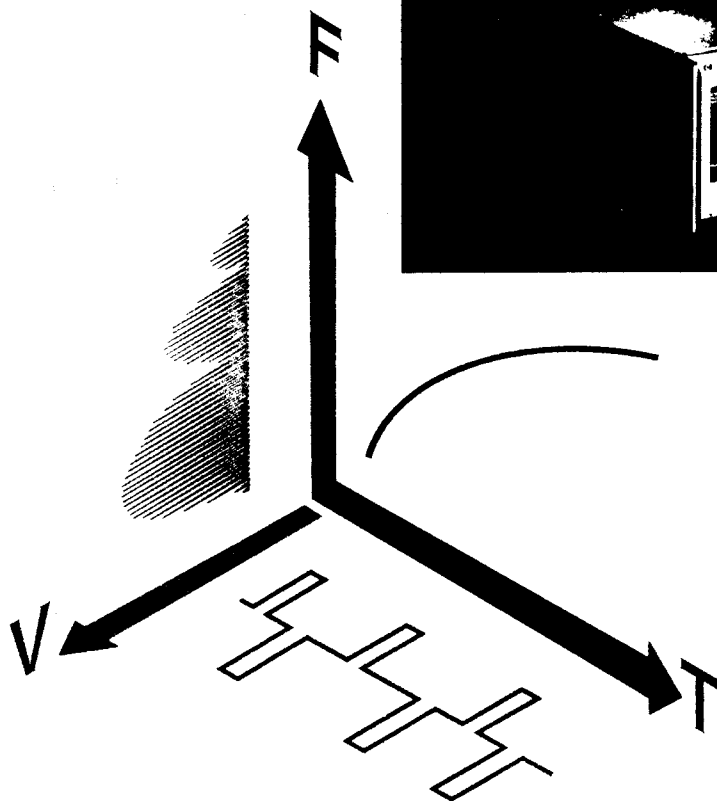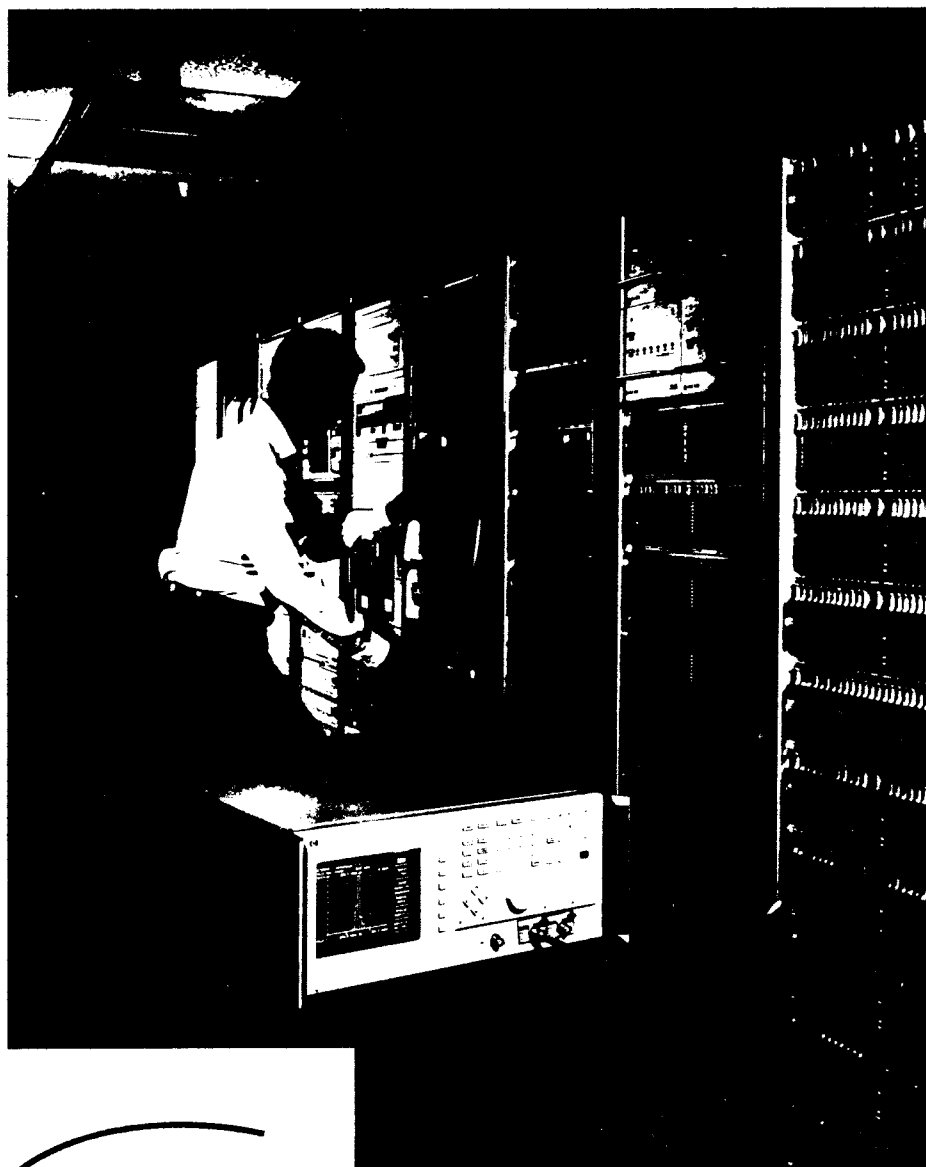
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INTRODUCTION

Jitter and wander are timing error phenomena encountered in every digital communications network (see *Figure 1*). Jitter refers to high-frequency error, usually defined as greater than 10 Hz. Wander, on the other hand, refers to low-frequency instability, observable only after 100 ms or longer. When a stream of digital pulses in a communications network exhibits excessive jitter or wander, these errors can severely degrade the network's performance in the transmission of information. The HP 5371A Frequency and Time Interval Analyzer, capable of making continuous time interval measurements on a stream of digital pulses (*Figure 2*), provides a powerful tool to help you capture and analyze these timing error phenomena.

The very nature of continuous time interval measurements gives you the flexibility and precision needed in analyzing jitter and wander. To familiarize you with the instrument's powerful measurement capabilities, this application note explains how continuous measurements, together with the HP 5371A's unique feature set, contribute to jitter and wander analyses. It also documents how to operate the instrument in making such measurements and the various ways of interpreting measurement results.

Specific measurements that this application note discusses include the following:

Histogram and Statistical Analysis of Data-to-Clock Jitter (*Figure 3*): a statistical distribution of data-to-clock timing stability. This analysis can show peak, mean and standard deviation of timing error amplitude, and so quickly characterize and predict the performance of communication devices in real-life network environments.

Jitter and Wander Spectrum (*Figure 4*): spectral content of the phase modulation on a pulse stream. This analysis is useful when there are identifiable sources of interference, such as power line jittering data or clocks. By recovering jitter or wander as phase modulation on a carrier and performing a Fourier spectral analysis with the HP 5371A and a computer, you can investigate the various causes of unexpected timing errors.

Also, with the capability of measuring jitter on both data and clock, the HP 5371A is an excellent tool for characterizing the transfer function of multi-port devices such as phase-locked loops, clock-recovery circuits and other digital network interfaces.
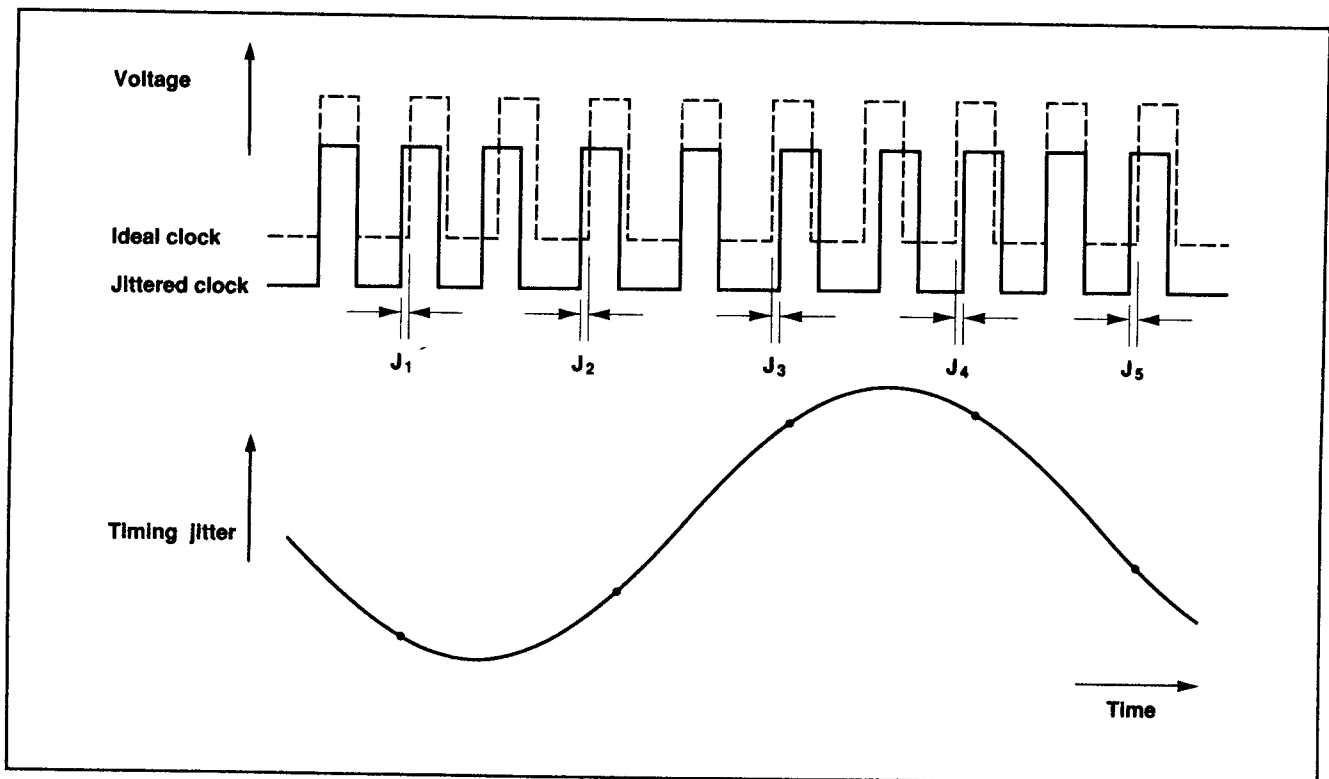

Figure 1. Jitter is the effect of the real clock edges varying in time relative to an ideal (stable) clock. The HP 5371A can directly measure these times $J_1, J_2 ... J_n$ to precisely characterize jitter.

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Allan Variance (*Figure 5*): a measure of wander, or long-term frequency stability over time. A log-log plot of Allan variance vs. measurement-averaging time can reveal the sources of random noise in your device.
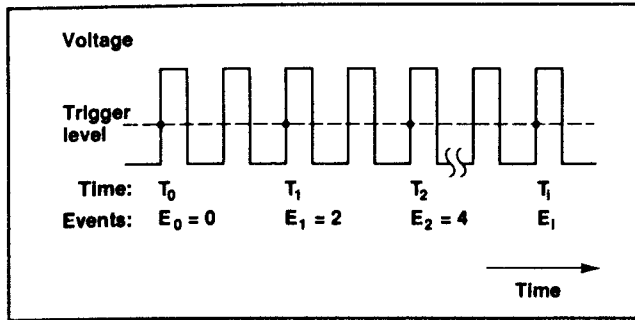

Figure 2. The technique employed by the HP 5371A is to precisely measure the times T_0 T_1... T_n to 150 ps rms resolution, and to record events E_0 E_1... E_n which can be occurring at rates to 565 MBits/s.
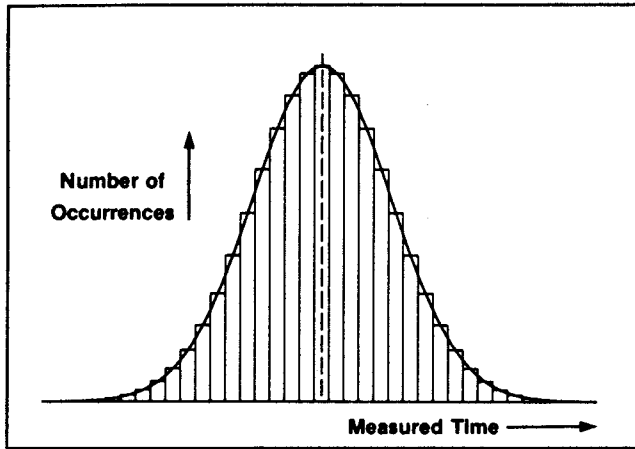

Figure 3. The histogram provides a powerful technique for the analysis of jitter. Once the distribution is characterized, predictions can be made about the probabilities of errors.
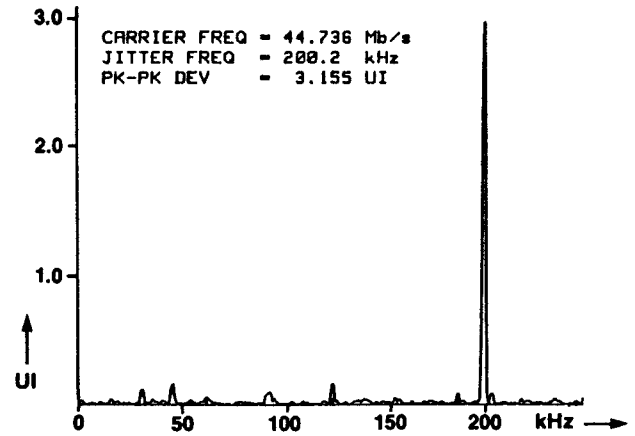

Figure 4. The spectrum of the jitter aids in understanding both whether the network will function error free, and perhaps what the source of the jitter is. This picture is an FFT of the data collected by the HP 5371A.
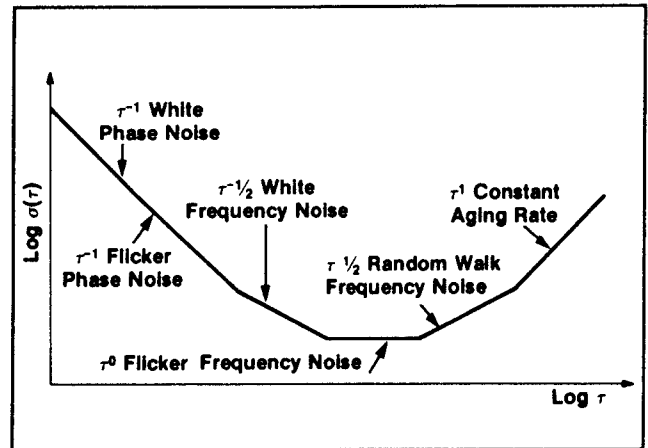

Figure 5. By observing the slope of the plot of log $\sigma(\tau)$ vs. log τ, you can determine the type of instabilities in your system clocks.

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HP 5371A'S MEASUREMENT CONTRIBUTIONS
TO JITTER AND WANDER ANALYSIS

Continous time interval measurements on the HP 5371A make it a high-performance, general-purpose jitter and wander receiver. Because of the instrument's unique feature set, the HP 5371A makes the following valuable contributions to jitter and wander analyses:

Flexible Input Clock Rates to 500 Mb/s*. The HP 5371A's 500 MHz input bandwidth accepts input data or clock with rates up to 500 Mb/s, making it a practical general-purpose tool for timing analysis. Jitter and wander can be characterized on not only traditional DS1 (1.544 Mb/s), DS2 (6.312 Mb/s) and DS3 (44.736 Mb/s) (or CEPT 2.048, 8.448, 34.368 Mb/s) signals but also signals of higher transmission hierarchies or pulse streams of other data rates.

150 ps Jitter Sensitivity. Measuring edge jitter of digital pulses with 150 ps rms time interval resolution, the HP 5371A has exceptional sensitivity to timing jitter. For instance, for DS1 signals, the HP 5371A can detect timing deviation smaller than 0.03% of a 647 ns bit window or 0.0003 unit intervals (UI). This resolution results in accurate measurements on jitter amplitude.

Detection of Large Jitter or Wander Amplitude. Because the HP 5371A uses a time tagging technique to measure jitter, it can detect wide timing deviations and, therefore, places no constraints on the jitter or wander amplitude, or range of input clock frequencies.

Capture of Jitter Spectral Content to 5 MHz. With 10 Ms/s continuous measurement rate, the HP 5371A can demodulate timing errors with spectral content as high as 5 MHz. It is important to be able to analyze high-frequency jitter because a small amount of it can be as disruptive to information transmission as a large amount of low-frequency jitter.

Wander Characterization. With one instrument, you can characterize both short-term timing jitter as well as long-term frequency wander. The HP 5371A's built-in frequency and Allan variance computational algorithms automatically give you the fractional stability results you need. An internal high-stability oven timebase provides short-term frequency stability better than 1×10^{-10} for 1s averaging and long-term stability better than $5 \times 10^{-10}/$ day. To measure wander of more stable clocks, it is possible to synchronize the instrument's timebase with a cesium standard via the timebase input. In making long-term stability measurements, you no longer need to generate non-standard reference frequencies from a time standard.

Flexible Analyses. As a general-purpose jitter and wander receiver, the HP 5371A detects edges of digital pulses and digitizes their timing and phase information. Via HP-IB, you can transfer the recorded data to a computer for any customized processing you need. As illustrated in the next few sections of this application note, from the raw data, you can recover the phase modulation on the clock, view statistical distribution, examine spectral content, or plot fractional frequency stability as a function of time and much more. This flexibility makes the HP 5371A an excellent general-purpose tool for analyzing jitter and wander.

* By special request, 565 Mb/s can be accommodated.

STATISTICAL ANALYSIS OF DATA-TO-CLOCK JITTER ███████████

Objective

By making time interval measurements from live data traffic to a system clock (*Figure 6(a)* and *(b)*) and viewing the relative timing jitter in the form of statistical distribution, the HP 5371A can help you predict the error-rate performance of digital communication networks. The assumption made here is that the clock is jitter free. See page 7 for a method to verify and characterize clock jitter.

Instrument Setup

a) Connect the data and clock signals to the HP 5371A as shown in *Figure 6(a)*.

b) Adjust input trigger-level conditions: SINGLE AUTO TRIGGER, 50% of peak-to-peak amplitude; positive crossings (POS) for both CH A and CH B (for bipolar signals, set to 75% of peak-to-peak).

c) Select measurement function: TIME INTERVAL, A → B (*Figure 7*).

The HP 5371A offers a variety of measurement functions, FREQUENCY, PERIOD, PHASE, PULSE-WIDTH, TIME INTERVAL, to name a few. As shown in *Figure 6(b)*, TIME INTERVAL will let you measure the time intervals from a stream of data edges (CH A) to clock edges (CH B).

d) Select arming/triggering mode: AUTOMATIC SAMPLING.

There are a variety of arming and triggering modes available when making time interval measurements. Select AUTOMATIC SAMPLING, which will start a block of measurements as soon as the instrument is ready and make A → B time interval measurements at the instrument's maximum 2-channel measurement rate: 5 M samples/s. Note that if input data is coming in at a rate less than 5 Mb/s, the HP 5371A will be able to capture the timing jitter on every edge. If the data rate is higher than 5 Mb/s, some edges will not be measured. Missed edges do not present a measurement problem because data-to-clock jitter, mostly attributed to random noise, can be characterized with a large number of samples, and generally is less than 1 MHz in bandwidth.

e) Specify measurement block size and number of blocks: 10 blocks of 1000 measurements each.
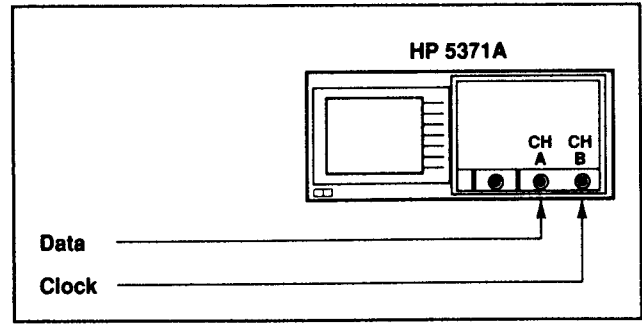
(a)
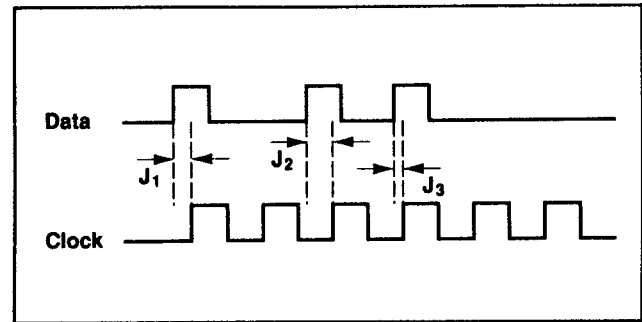
(b)

Figure 6(a) and (b). The HP 5371A measures jitter directly by measuring the time from data edge to clock edge.

Figure 7. The HP 5371A is set up to make 10 blocks of 1000 measurements between channel A (data) and channel B (clock). The arming is automatic; measurements are made as soon as the instrument is ready.

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You can choose a block size N less than 1000*, the maximum capacity the stand-alone HP 5371A offers for post-analysis. You can specify multiple blocks of N measurements to increase the confidence level of the statistical analysis to be performed. For instance, 10 blocks of 1000 measurements will yield all together 10,000 measurements for cumulative statistical calculations and histogram analysis.

f) Turn on the statistics function in the MATH MENU: STATISTICS ON.

g) In the GRAPH FORMAT MENU, select UPDATE mode for histogram plot: UPDATE WHILE or AFTER.

In the WHILE mode, the cumulative histogram will be updated at the end of each measurement block. In the AFTER mode, the histogram will be plotted only once after all the measurement blocks are completed. For throughput considerations, AFTER is the preferred mode.

Histogram Analysis

The HP 5371A's built-in histogram plot provides a graphical and statistical representation of timing jitter, in which the data-to-clock time intervals are sorted according to their values. As shown in *Figure 8*, the range between minimum and maximum jitter is divided into a number of equal subranges or bins. Each of the 10,000 jitter measurements is placed in the histogram by determining which bin it falls into and then incrementing the corresponding bin count by one.

By doing this sorting automatically on the instrument, the HP 5371A can display results like *Figures 9* and *10* on the built-in CRT. A plot from the GRAPHIC mode, *Figure 9* shows a Gaussian jitter distribution for a DS1 signal. The distribution's mean and standard deviation are displayed on the NUMERIC display as shown in *Figure 10*, or by turning STATISTICS ON in the HISTOGRAM display.

* If you are using the instrument without a controller, 1000 measurements per block are available for analysis. However, up to 4095 measurements are accessible via HP-IB when the instrument is used with a computer.
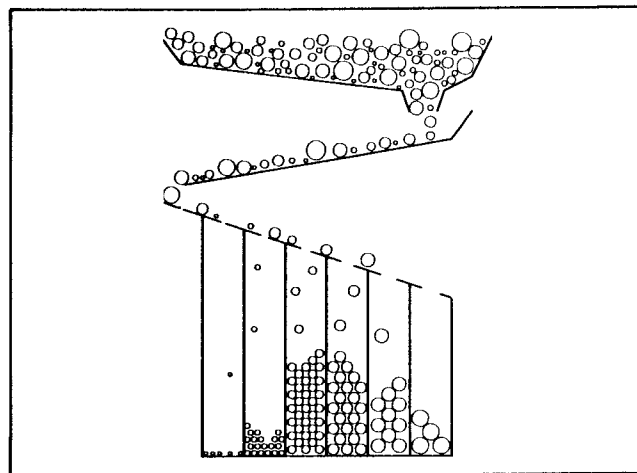

Figure 8. Here is a pictorial view of how histograms are built. Measurements are sorted by size into the first bin large enough to take them.

Statistics based on 10000 measurements.
Time Int A -> B [12 Oct 1987 16:50:32]
Y Axis max count: 949 meas
Min: 295.4 ns   Max: 410.2 ns

X Axis: 102.6 ns   to 502.6 ns
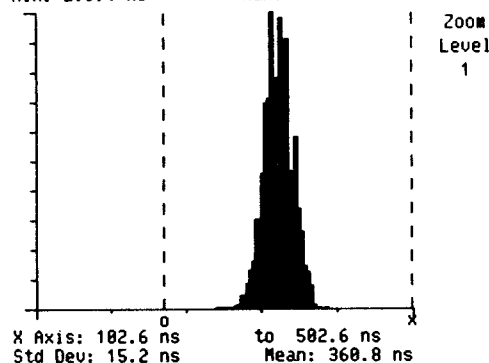Std Dev: 15.2 ns   Mean: 360.8 ns

Figure 9. This is the distribution of a stable DSI signal relative to a synthesized clock. It is plotted directly on the display of the HP 5371A, with statistics if selected.

STATISTICS DISPLAY	
Time Interval A -> B	12 Oct 1987 16:50:32
Block # 10	10000 Measurements
Mean	360.794 ns
Std Dev	15.195 ns
Maximum	410.0 ns
Minimum	295.6 ns
RMS	388.691 ns
Rt Al Var	8.939 ns
Allan Var	79.902 822E-18s^2
Variance	230.881 776E-18s^2

Figure 10. The statistics of the measurements can also be displayed numerically.

Error Ratio Prediction

In addition to convenient visual analysis of jitter severity, the information obtained in the histogram plot is also useful for "margin analysis" or bit-error-ratio prediction. Knowing the statistical distribution of inherent data-to-clock timing error in live network traffic, a systems designer can now judiciously choose a data-detection timing margin to achieve the error-rate goal.

For instance, if data detection occurs at the rising edge of the clock as shown in *Figure 11(a)*, one way to predict the bit error ratio (BER) is to characterize the distribution of the time intervals from the data pulses' rising edges to the clock's rising edges. A bit-detection error occurs when the clock's detecting edge falls out of the bit data pulse, with the time interval from data to clock either exceeding the data pulse width or falling below 0.

Such a distribution of data-to-clock time intervals often has characteristics of a normal distribution. By estimating its mean and standard deviation from a large number of measurements, you can predict its probability of detection error over a long period in a small amount of time. For instance, the time interval distribution of the system shown in *Figure 11(b)* appears normal, with a mean of 360.8 ns and a standard deviation of 15.2 ns. Given these estimations, the normalized distribution function can be expressed as the following:

$$p(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(t-m)^2}{2\sigma^2}}$$

You can calculate its probability of detection error PE on a computer by integrating the probability of time intervals exceeding the pulse width or falling below 0.

$$PE = \int_{t=-\infty}^{0} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-m)^2}{2\sigma^2}} dt +$$

$$\int_{t=PW}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-m)^2}{2\sigma^2}} dt$$

To simplify computation of PE, translate the variable within the integral with $t' = (t-m)/\sigma$ and use a standard error function table for computation:

$$PE = \int_{t'=-\infty}^{-360.8/15.2} \frac{1}{\sqrt{2\pi}} e^{-\frac{t'^2}{2}} dt' +$$

$$\int_{t'=(PW-360.8)/15.2}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t'^2}{2}} dt'$$

By performing the above Gaussian estimation and integration, a system engineer designing the network can now vary the data pulsewidth and run a margin analysis. A plot showing PE as a function of pulsewidth can graphically present the tradeoff between BER and data rate.
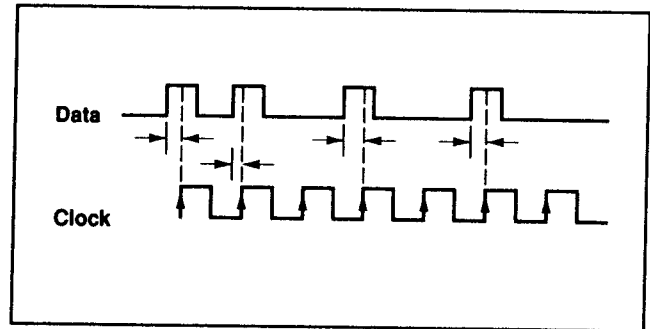
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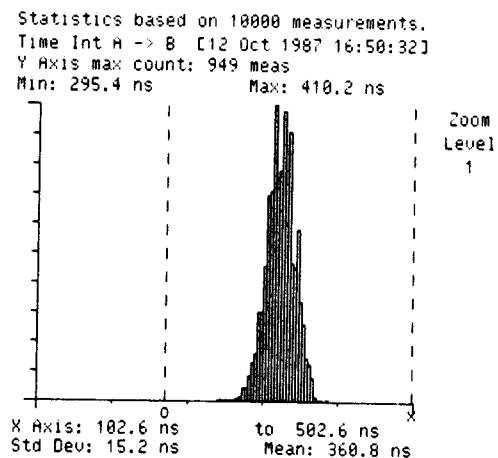
(b)

Figure 11. Plotting a histogram of where clock edges fall with the data pulse allows prediction of the probability of the clock falling outside the data pulse. This is, therefore, a prediction of error ratio.

JITTER/WANDER DEMODULATION OF A CLOCK
- JITTER HISTOGRAM AND SPECTRUM ▬▬▬▬▬▬▬▬

Objective

Data-to-clock jitter, as illustrated in the previous section, is a useful measure of relative timing stability between live data and the system clock. However, it does not detect the inherent jitter that both data and clock may have simultaneously. This inherent instability, though seemingly harmless within the subsystem, may cause significant problems in network interfaces.

The objective of this section is to analyze clock jitter and wander inherent in every communications network by treating these timing error phenomena as phase modulation (PM) on a carrier and use the HP 5371A and a computer to demodulate PM on a stream of clock pulses. A spectral analysis of the recovered jitter or wander can yield valuable information about the severity and causes of these timing errors.

General Theory

Jitter and wander are pulse timing errors that can be mathematically modelled as phase modulation J(t) on a clock carrier (see *Figure 12*):

$$V(t) = A \sin(2\pi f_c t + J(t))$$

$$\phi(t) = 2\pi f_c t + J(t)$$

V(t) is the voltage representation of the carrier with modulation; $\phi(t)$ is its cumulative phase relative to a starting pulse. For simplicity, this model treats only the fundamental frequency f_c of the digital clock as the carrier whose phase J(t) modulates.
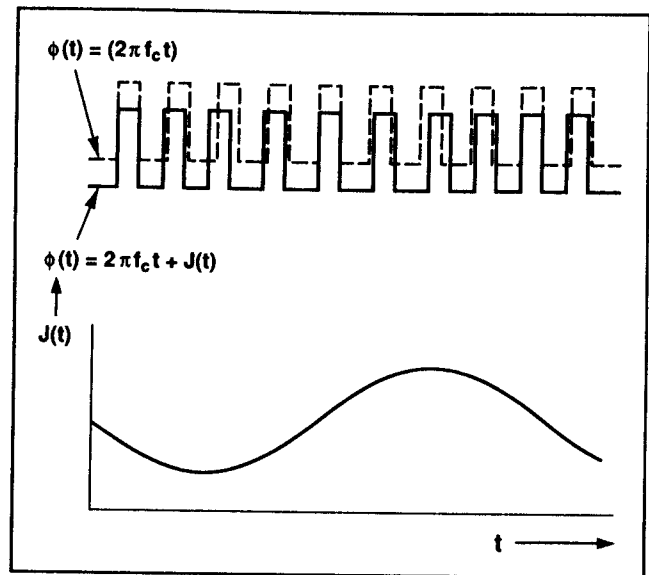

Figure 12. J(t) is a plot of the sampled jitter; the time interval between the measured clock edge positions is real time vs. the ideal clock.

Demodulating J(t) with the HP 5371A and a computer is a three-step process (see *Figure 13*).

(1) Digitize the signal's cumulative phase $\phi(t)$ (see *Figure 13(a)*).

In capturing jitter or wander, the HP 5371A functions like an instantaneous phase digitizer. At user-specified* sampling intervals, the HP 5371A detects the instantaneous phase of a stream of digital pulses relative to a starting pulse. As shown in *Figure 2* (Page 2), it does this by totalizing the number of pulses, each of which represents a phase addition of 2π, and recording the exact time at which the totalize measurement is made. By saving the changing phase information as a function of time in internal memory, the HP 5371A effectively captures the timing perturbation on the clock carrier.

(2) Estimate fundamental carrier frequency f_c (see *Figure 13(b)*).

Given $J(t) = \phi(t) - 2\pi f_c t$, to recover $J(t)$ from $\phi(t)$ requires an estimate of f_c, the fundamental frequency of the digital clock, from the digitized $\phi(t)$ data. As shown in *Figure 13(b)*, this is equivalent to finding the "average" slope of $\phi(t)$. To perform this estimation, one of several algorithms can be used. The two-point method as illustrated in *Figure 14* is a simple example. In the next section, a more sophisticated estimation algorithm will be recommended in a measurement example.

(3) Recover phase modulation $J(t) = \phi(t) - 2\pi f_c t$ (see *Figure 13(c)*).

$J(t)$ can be conceptualized as phase deviations that a carrier exhibits as a result of modulation. $\phi(t)$ represents the pulse stream's actual phase. $2\pi f_c t$ signifies the phase of the ideal, unmodulated carrier. $J(t)$, timing jitter and wander, or phase perturbation, is the difference between the two.

Instrument Setup

To help you get started quickly in making jitter measurements on a stream of pulses, this section serves as an instrument-setup guide to lead you through the few simple steps of getting the HP 5371A ready for a measurement. Although the measurement objective is specific - to sample and record the timing perturbation, bandlimited to 800 kHz, on a DS1 (1.544 MHz) - the general procedure should assist you in making other measurements on similar sources as high as 500 MHz.
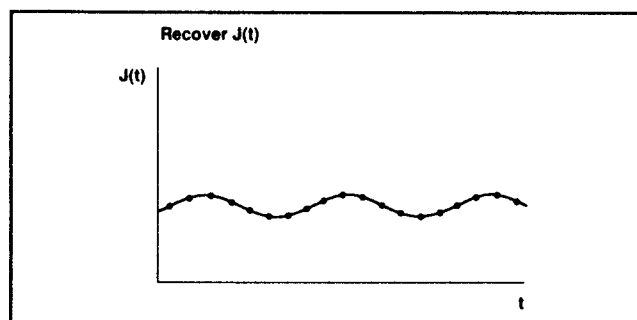
Note that sampling actually occurs synchronized with the input signal edges. Thus, the user selected interval is only approximate, but the measured time is precise.
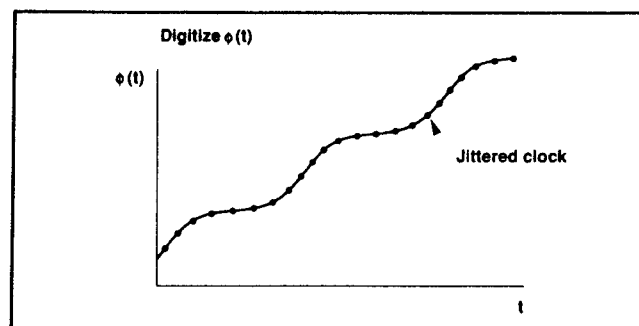
(a)
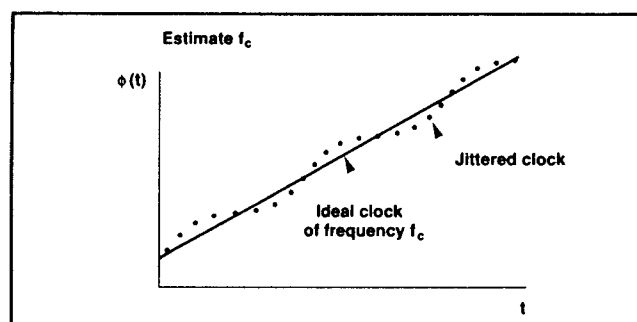
(b)

(c)

Figure 13. *Extracting jitter with the HP 5371A is a three step process. (a) Measure the signal's cumulative phase, (b) estimate the carrier frequency, (c) subtract the phase variation from the recovered clock.*
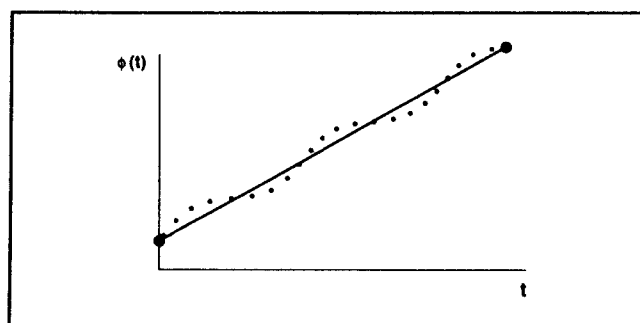

Figure 14. *The simplest way to estimate the clock frequency is to connect the end points of the phase data gathered by the HP 5371A. A least squares fit or two centroid method could also be applied.*

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a) Connect the clock under test, the HP 5371A and an HP-IB-compatible computer as shown in *Figure 15*.

To recover the jitter modulation on the input clock, you need an HP-IB-compatible computer to control the HP 5371A in making measurements and analyses. You can use either an HP 9000 Series 200/300 controller or an IBM PC/XT/AT-compatible computer with an HP-IB interface card.
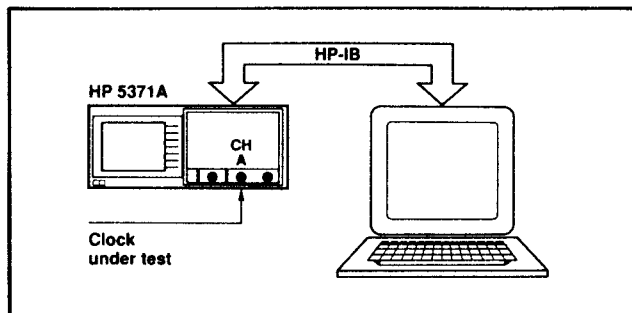

Figure 15. The HP 5371A is fully programmable and can transfer measurement results very rapidly (up to 20,000 measurements/sec) to the computer.

b) Program the trigger level for continuous time interval measurements: SINGLE AUTO TRIGGER, 50% of peak-to-peak amplitude.

c) Program measurement function - **CONTINUOUS TIME INTERVAL** (see *Figure 16*)

The HP 5371A Frequency and Time Interval Analyzer offers a variety of measurement functions: FREQUENCY, PERIOD, PHASE, CONTINUOUS TIME INTERVAL, to name a few. For jitter demodulation, CONTINUOUS TIME INTERVAL, a single-channel measurement, will be the primary focus.

Figure 16. Set up this way, the HP 5371A records the time at which the next clock edge occurs at a delay of 600 ns from the last measurement. Thus, phase is sampled at a 1.66 MHz rate.

d) Program Arming and Trigger Mode -- **INTERVAL SAMPLING, 600 ns INTERVAL**

There are a variety of arming and triggering modes available in CONTINOUS TIME INTERVAL measurements. As shown in *Figure 17*, the mode you select controls when the HP 5371A starts a block of measurements as well as when each measurement within the block is made. Select INTERVAL SAMPLING, which will start a measurement block as soon as the instrument is ready and make measurements at a user-specified interval.
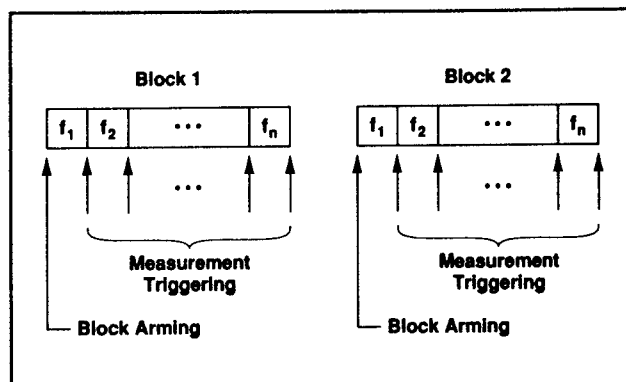

Figure 17. The interval set in INTERVAL SAMPLING mode is a delay from one measurement to being ready to measure the next edge. Here we are using AUTOMATIC block arming.

To determine the appropriate sampling interval, first determine how fast you need to make continuous time-interval measurements. In general, the measurement rate required by your device depends on its jitter spectral content: Nyquist's Sampling Theorem requires that you sample the timing or phase modulation at least twice as fast as the highest frequency of the jitter. For 800 kHz-bandlimited jitter on the DS1 clock under test, a sampling rate of 1.666 MHz (600 ns sampling interval) is sufficient.

If the bandwidth of the jitter spectrum is not known, as is often the case when testing the clock for the first time, select AUTOMATIC SAMPLING mode instead of INTERVAL SAMPLING. This arming mode will automatically set the HP 5371A's measurement rate to 10 Ms/s, its fastest. According to Nyquist's Sampling Theorem, this phase sampling rate will let you analyze phase modulation spectrum to 5 MHz. Of course, the spectrum must be bandlimited to less than 5 MHz to avoid undesired aliasing.

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In general, you should run a first-pass spectral analysis on the data taken in AUTOMATIC SAMPLING mode. Should the results show a narrower jitter spectrum than 5 MHz, you can then switch the arming mode to INTERVAL SAMPLING and choose a slower sampling rate as appropriate. A slower sampling rate will improve the spectral resolution as explained in (e) below. Note that in INTERVAL SAMPLING, the sampling interval is variable from 600 ns to 8s.

e) Program measurement block size and number of blocks: **1 block of 4095 measurements**

You can choose a block size N up to 4095. In general, N directly affects the frequency resolution of the jitter spectrum to be obtained:

$$Frequency\ Spectral\ Resolution = \frac{Sampling\ Frequency}{N}$$

Note that the larger N is, the longer the measurement and computation time, but the better jitter spectral resolution. For the HP 5371A to capture wander error components in addition to jitter, this resolution needs to be finer than 10 Hz.

f) Program Data Format for HP-IB Transfer BINARY

To recover jitter or wander from the phase information that the HP 5371A has digitized, you need to transfer the recorded data to the computer via HP-IB. Set data format to BINARY to achieve the instrument's up to 20,000 measurements/s HP-IB transfer rate.

Now you are ready to make a single block of measurements.

Jitter and Wander Recovery

Once the instrument's raw data is transferred to the computer via HP-IB, demodulate jitter and wander from it as explained on page 8.

a) Collect Raw Data E(t)

Figure 18(a) shows you data the HP 5371A collects for post-processing. At regular sampling intervals, the instrument looks for the next trigger-level crossing to make a measurement. For each measurement, the instrument records two pieces of information: E_i, the number of trigger-level crossings (or clock pulses in this example) that have occurred since the instrument was first armed; and T_i, the precise time at which the crossing count is taken. E_i represents phase information,

with each crossing signifying a phase addition of 2π ($\phi_i = 2\pi E_i$). The combination of E_i and T_i shows how the clock's phase varies as a function of time (*Figure 18(b)*).

b) Estimate Clock Frequency f_c

To estimate the clock frequency f_c from the digitized phase data E_i and T_i, the following algorithm is recommended:*

$$f = \frac{\sum_{i=1}^{M}\left(E_{i+2M} - E_i\right)}{\sum_{i=1}^{M}\left(T_{i+2M} - T_i\right)}$$

M = N div 3

E = Cumulative number of trigger-level crossing for the ith measurement

T = Time at which E_i is counted

This algorithm is simple and fast and only 6% worse than a mean square error fit.

For instance, if you select 1000 measurements (N = 1000), then M = 333. Note that in demodulating the

Events	Time ns
1	0
2	647.0
3	1294.0
4	1940.4
5	2587.6
6	3234.4
7	3881.6
8	4528.4
9	5176.0
10	5823.2

(a)
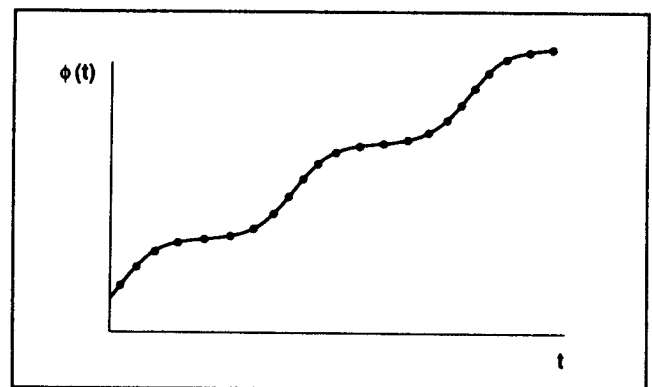
(b)

Figure 18(b). When plotted, we have the phase samples from which jitter can be extracted.

jitter and wander on your clock, you also get an accurate measurement on the clock frequency.

c) Compute Jitter and Wander J(t)

You can demodulate jitter and wander in units of either time (seconds), fraction of a clock period (unit interval or UI), or phase (radian):

Second: $J_i = (E_i - E_o) / f_c - (T_i - T_o)$

UI: $\quad J_i = [(E_i - E_o) - f_c (T_i - T_o)]$

Radian: $J_i = 2\pi [(E_i - E_o) - f_c (T_i - T_o)]$

$\qquad i = 0$ to 4095

Jitter and Wander Analysis

With jitter J(t) recovered from the measurement results, you can now analyze its severity and causes in two ways:

a) Jitter amplitude as a function of time (*Figure 19(a)*)

Figure 19(a) is such an example, showing the recovered jitter (200 kHz-bandlimited) of a DS1 clock. It tells you the jitter amplitude, with minimum and maximum timing jitter at ~ +100 ns and ~ - 90 ns respectively; the peak-to-peak range of 190 ns is equivalent to about 0.28 UI (190 ns/647 ns). The figure also shows a clear periodicity in the varying jitter amplitude. A quick glance suggests that it's approximately 120 Hz, an indication that the source of interference is the power line. This can be quantified by using the cursors.

Indeed, with better shielding implemented in the design of the clock generator, a new measurement results in *Figure 20* (next page). Timing instability is now considerably reduced. The peak-to-peak jitter, which now appears dominated by a higher frequency component, is only about 30 ns or 0.05 UI.

b) Jitter and Wander Spectral Analysis (*Figure 19(b)*)

You can also analyze jitter and wander by examining their spectral content. A spectral analysis is extremely useful when there are multiple sources of interference jittering the data clock. It helps you identify and eliminate these interferences individually.

To do this, perform a Fast Fourier Transform (FFT) on J(t), the recovered jitter, and then plot the resulting spectrum on the computer. The appendix documents some special considerations in performing the transform.
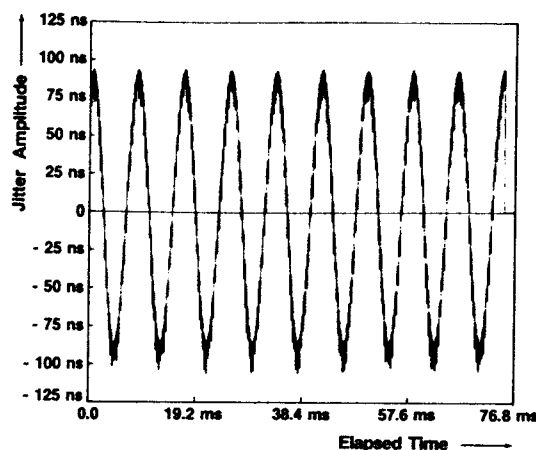

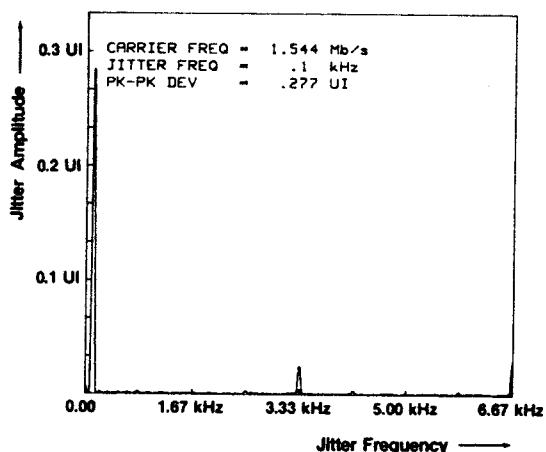*Figure 19(a). A 120 Hz power line related jitter component is apparent in the analysis of this clock circuit.*

Figure 19(b). A transfrom to the frequency domain shows more clearly the frequency and magnitude of this and other jitter components.

Figure 19(b), a spectral analysis of the 120 Hz power line jittering the DS1 clock, is obtained from *Figure 19(a)* through an FFT. The x-axis of the spectrum shows jitter frequencies; the y-axis shows the peak-to-peak jitter amplitude in seconds, UI, or radians at the corresponding frequencies. Note that a wide jitter dynamic range is achievable by the HP 5371A: the minimum detectable timing error can be as low as the instrument's 150 ps rms time interval resolution; the maximum jitter allowed is limited only by memory.
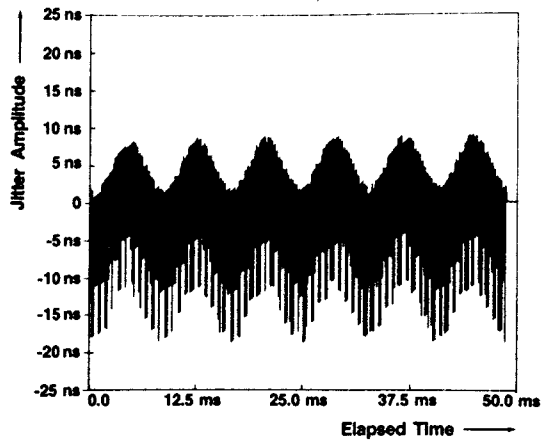
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Figure 20. Better design of the clock significantly reduces the 120 Hz component and shows more clearly a high frequency, low amplitude element of jitter.

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DEMODULATION OF JITTER ON DATA ■

Objective

The previous section showed you how the HP 5371A, when used with a computer, can demodulate jitter on a clock and analyze its spectrum. This section will discuss similar measurements and analysis, but now on a stream of data. Because the instrument's setup procedure for demodulating data jitter is similar to that for clock jitter, refer back to pages 8-11 for the setup procedures. Also, for jitter recovery and spectral analysis, you can use these pages for reference.

General Theory

The concept of phase sampling, as explained in clock jitter demodulation, is also applicable in demodulating data jitter. In fact, you can model jitter $J(t)$ exhibited by a stream of pseudo-random data as phase modulation on a clock $V(t)$ of the same data rate f_c (see *Figure 21*):

$$V(t) = A \sin (2\pi f_c t + J(t))$$

The method for extracting data jitter is similar to that for clock, except that the samples of phase are irregularly spaced (only at data "one"s). The problem then, assuming that we still sample frequently enough, is to discover which clock cycle we are comparing this data bit position to. This means taking one extra computational step.

1) Measure the input data edges' times of occurrence with the HP 5371A's time tagging continuous time interval mode (*Figure 22 (a)*). Each edge will be placed in time, relative to the first edge measured, with a resolution of 150 ps rms.

2) Compute the clock cycle number that this data bit corresponds to (*Figure 22 (b)*) using the following methodology. (It is assumed that an initial estimate of the clock period T is available.)

Starting at the first data bit at time t_1, we label this as clock cycle 1. The next data bit, measured as occurring at time t_2, is then computed to be $(t_2 - t_1)/T$ clock cycles later. This will be $n_1 T + J_1$ cycles, where n_1 is an integer, and J_1 is the jitter. Since over the short sampling intervals selected, J_1 will be $<< 0.5$ clock cycles, there will be no ambiguity as to which is the appropriate clock cycle for comparison.

Next, take the third data bit time t_3. To find its clock cycle number, find n_2 where $n_2 T + J_2 = (t_3 - t_2)/T$. Now its position is clock cycle number $n_1 + n_2$.

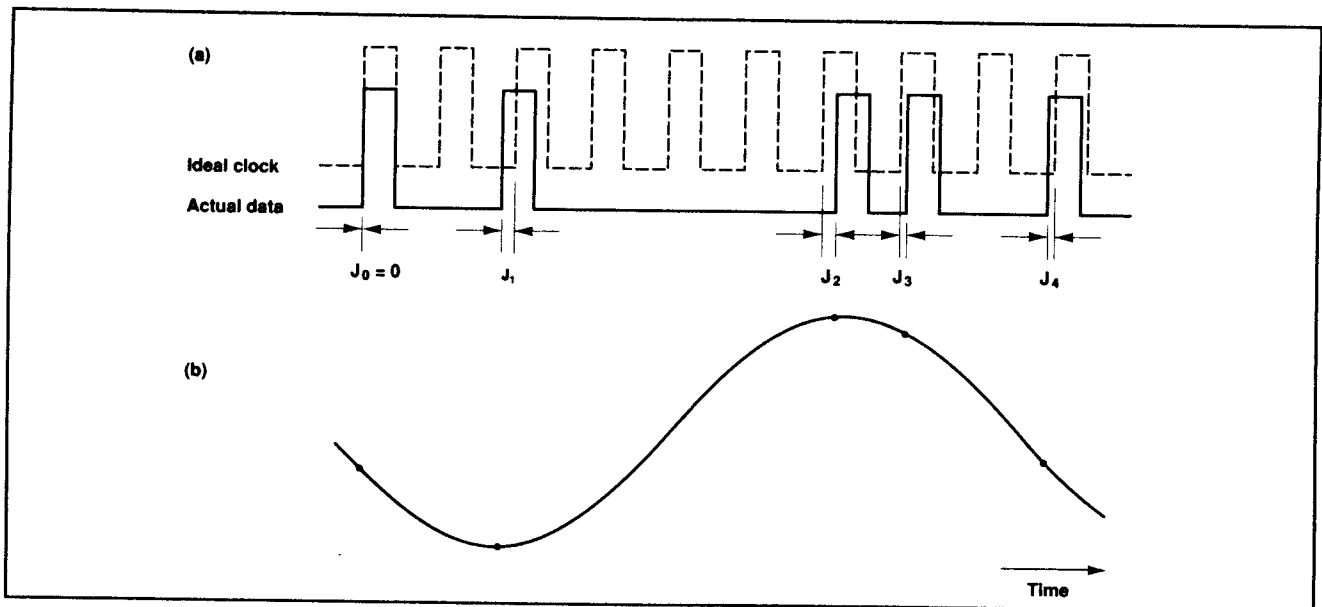Repeat this process for all collected edges.

Figure 21. Measuring jitter of a data signal follows the same basic principles outlined so far. The major difference is that the samples are not regularly spaced. This causes little problem provided we continue to sample frequently enough.

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3) We now have the data to reconstruct the phase plot shown in *Figure 22 (c)*. Note its similarity to the jittered clock case in *Figure 13*, the only difference being the irregular sampling. We can now proceed in the same manner as the clock case to estimate the average frequency *(Figure 22 (d))*, and thus extract the jitter *(Figure 22 (e))*.

Data Jitter Analysis

Data jitter can be analyzed in much the same way as in analyzing clock jitter - in both time and frequency domains (see pages 7-12). In the time domain you can plot jitter amplitude as a function of time and quickly view the peak jitter. You could plot a histogram over many measurements and compute the statistics. Finally, you could perform an FFT on the recovered jitter and view the jitter in the frequency domain. The spectral content could of course give you clues as to the potential sources of the jitter.

Also please see the appendix for considerations in transforming non-uniformly sampled data.
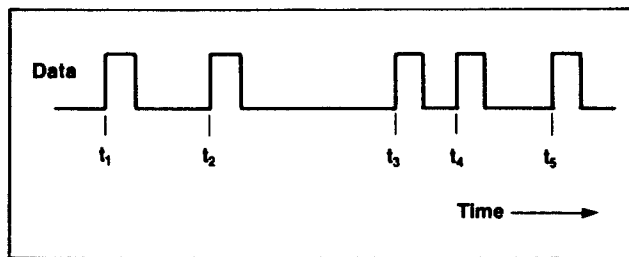
(a)
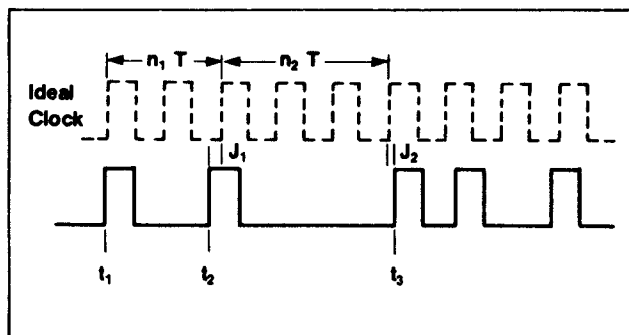
(b)
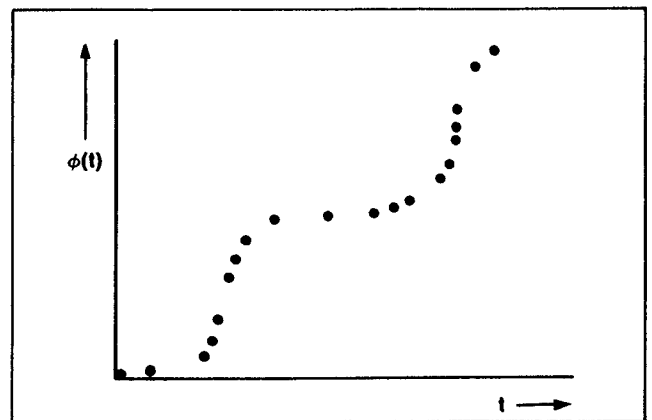
(c)
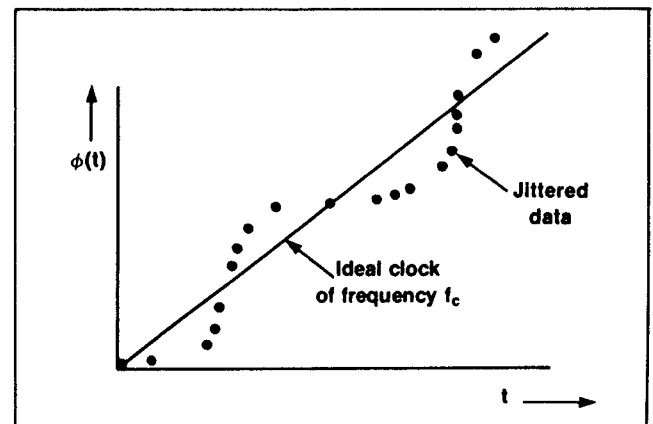
(d)
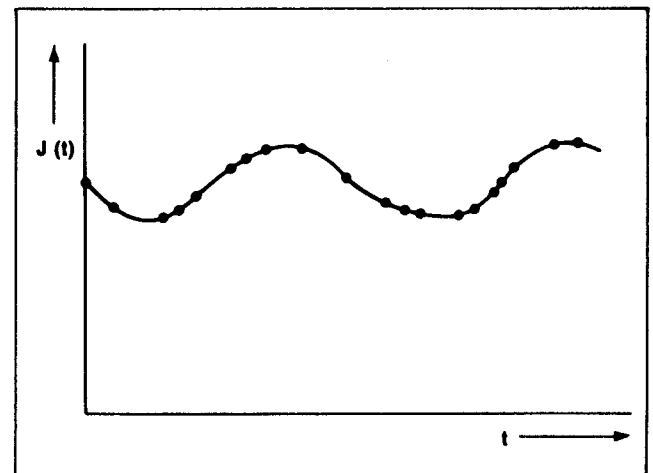
(e)

Figure 22. (a) Shows the data edges time tagged with the 150 ps rms resolution of the HP 5371A. (b) Shows the basic jitter extraction methodology of first determining which clock edge to compare the data bit with, and then noting its jitter. (c) Shows how the resultant samples of phase (each clock being a multiple of 2 π) plotted versus time. (d) Shows how the clock frequency can be extracted, and finally (e) shows the recovered jitter function.

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Jitter Transfer Function

Capable of measuring both clock and data jitter, the HP 5371A is a useful tool for characterizing the transfer functions of multi-port communication devices. For instance, for clock-recovery circuits (see *Figure 23*) or phase-locked loops, by analyzing the jitter on both incoming data pulses and outgoing clock cycles, you can characterize the performance of the devices in operating environments as well as under stressed conditions. Similar measurements can characterize network interfaces between different hierarchies (see *Figure 24*): DS1 and DS3, DS3 and devices of higher data rates, and much more. A comparison between the spectra of output and input jitter reveals the inherent limitations of the device.
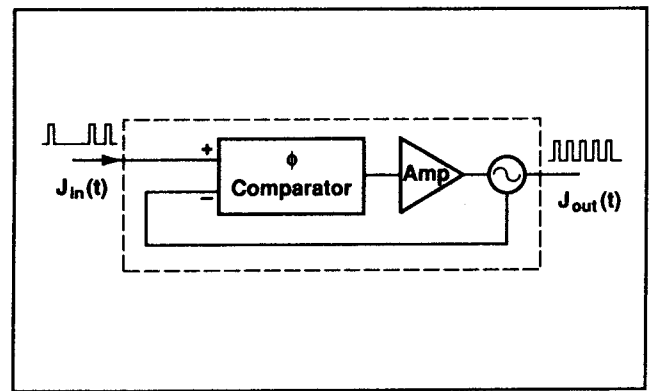

Figure 23. Clock recovery circuits can be characterized for jitter transfer function with relative ease, even using live data.
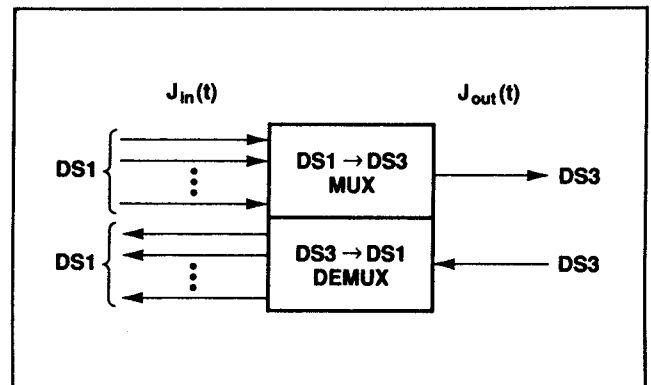

Figure 24. Because the technique employed by the HP 5371A is not clock rate dependent, correlations across a multiplex can be observed.

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CHARACTERIZATION OF CLOCK WANDER ■■■■■■■■

OBJECTIVE

To characterize wander, the HP 5371A makes continuous (zero dead time) measurements of frequency. The most straightforward analysis method is to use the technique described on pages 7 and 8 (and in *Figure 13* in particular). The HP 5371A can take precise measurements of Events (phase) and Time, every 100 ms for example, and continuously output its registers to a computer. From this data the recommended plot of Time Interval Error (TIE) and a calculation of Maximum TIE (MTIE) is easily made for any extended period of time.

Currently favored is the analysis of clock wander, or stability, by measurement of Allan variance vs. gate time. This is particularly effective with the HP 5371A on three counts. First of all the measurements of frequency are made with zero dead time (the original concept of Allan variance). Secondly, the calculation can be performed automatically in the instrument. And finally the measurement can be made much faster by virtue of the contiguous frequency measurements: for example the data collected for 1000 measurements of frequency at 10 ms gate time is the same as 100 measurements at 100 ms gate time which is the same as 10 measurements at 1s gate time.

This section demonstrates how to use the capabilities of the HP 5371A to measure a clock's fractional frequency stability, and examines a log-log plot of Allan variance ($\sigma(\tau)$) vs. frequency averaging time which can reveal the sources of random noise in a clock.

General Theory

Allan variance $\sigma^2(\tau)$ is defined as the average variance of two zero-dead-time frequency samples:

$$\sigma^2(\tau) = \left\langle \left(\sum_{i=1}^{2} \overline{f_i} - \sum_{j=1}^{2} \overline{f_j} \right)^2 \right\rangle = \frac{1}{2} \left\langle \left(f_1 - f_2 \right)^2 \right\rangle$$

τ is the gate time during which average frequencies f_1 and f_2 are measured, with no dead time between measurements.

The notation for average $\langle \rangle$ implies infinite measurement duration. However, in practice, $\sigma^2(\tau)$ can only be estimated from a finite number of frequency measurements N. The estimator $\sigma^2(\tau)$ used in the HP 5371A is as follows:

$$\hat{\sigma}^2(\tau) = \frac{1}{2(N-1)} \sum_{i=1}^{N-1} \left(\overline{f_{i+1}} - \overline{f_i} \right)^2$$

This estimator of the Allan variance is itself a random variable, whose value approaches the true variance $\sigma(\tau)$ as $N \to \infty$. It is outside the scope of this note to discuss the variance of this estimator. However, you may consult Reference 1 for further detail. In general, the confidence level improves as N increases. For measurement and computation of Allan variance with the HP 5371A, N > 10 is recommended.

A log-log plot of $\sigma(\tau)$ vs. σ, regardless of what N is, yields valuable information about the noise present in your signal. Specifically, the slope of the curve (see *Figure 25(a)*) reveals the various types of noise affecting your clock: white noise phase modulation, "flicker noise" phase modulation, white noise frequency modulation (also known as random walk phase), or "flicker noise" frequency modulation. Knowing the type of noise in your signal also helps you compute the spectral density of the phase noise in the system. (Reference 1.)

Instrument Setup

a) Configure the test system as shown in *Figure 26*.

When used as a stand-alone, the HP 5371A computes both frequency and Allan variance for a given gate times and displays the results on the built-in CRT. However, to get a log-log plot of $\sigma(\tau)$ vs. τ, an HP-IB-compatible computer must be used.

b) Program trigger level: SINGLE AUTO TRIGGER, 50% of peak-to-peak amplitude.

c) Program measurement function: FREQUENCY

d) Program Arming/Triggering Mode: INTERVAL SAMPLING

In INTERVAL SAMPLING, the HP 5371A will start a measurement block as soon as the instrument is ready and make frequency measurements at a user-specified interval.

For $\tau <= 8s$, this sampling interval, variable from 600 ns to 8s with 100 ns precision, is equivalent to τ, the gate time during which average frequency is measured. The HP 5371A can make continuous frequency measurements and automatically compute Allan variance.

For $\tau > 8s$, the HP 5371A's maximum sampling interval, a computer is needed to compute both frequency and Allan variance from the instrument's binary data.

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For instance, for τ = 100s, set the sampling interval to 5s. Compute the average frequency for τ = 100s from 20 continuous frequency measurements with τ = 5s each:

$$f_{\tau=100} = \frac{E_{20} - E_0}{T_{20} - T_0}$$

E_i and T_i are cumulative phase (number of trigger-level crossings) and time which the HP 5371A measures at approximately 5s intervals. The slope between the two points (E_0, T_0) and (E_{20}, T_{20}) is the average frequency during 100s of gate time.

e) Program Block Size and Number of Blocks -- 1 block of N measurements.

Set the number of blocks to one for one block of N measurements.

For τ < = 8s, a block size N = 2 is adequate for computing two-sample Allan variance for the log-log plot. However, to increase the confidence level of the estimate of $\sigma(\tau)$ for frequency stability, a larger N for variance averaging should be used. In general, N > 10 is recommended. The larger it is, the smaller the variance of this estimator.

For τ > 8s, a block size N = 2M is adequate for computing two-sample Allan variance for the log-log plot, where M is the number of frequency measurements the HP 5371A needs to make to get one equivalent measurement with gate time τ. Again, for better estimate of $\sigma(\tau)$, N = 20M is recommended.

f) Turn on the Statistics function in the MATH menu: STATISTICS ON if you are not using BINARY output mode.

g) Program Data Format

When viewing frequency and Allan variance on the HP 5371A's built-in CRT, turn on the ASCII function. When using the instrument with a controller, especially for τ > 8s, turn on the BINARY function. Note, however, that with BINARY mode you must do your own calculation of Allan variance in the computer.

	Slope
White Phase Noise	-1
Flicker Phase Noise	-1
White Frequency Noise	-½
Flicker Frequency Noise	0
Random Walk Frequency Noise	½
Constant Aging Rate	1

(a)
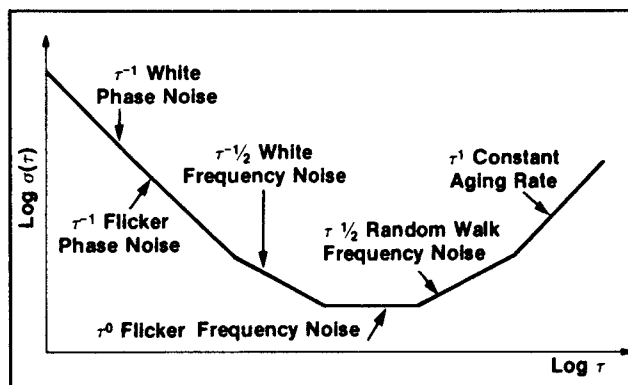
(b)

Figure 25(a) and (b). The slope of the observed variations of log $\sigma(\tau)$ vs. log (τ) indicates the type of instabilities in a clock source.
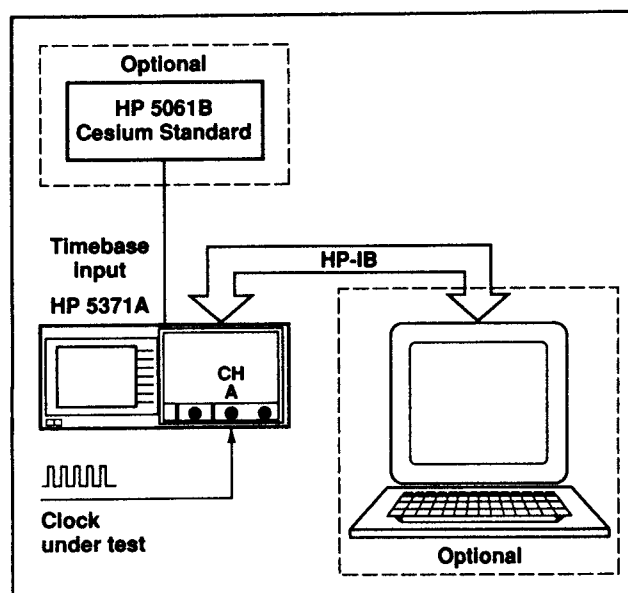

Figure 26. With a much simplified set up compared to currently available techniques, the HP 5371A can easily quantify frequency stability. For stabilities approaching parts in 10^{10}, use a cesium standard for the HP 5371A timebase.

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h) Program Math Function (see *Figure 27*)

For fractional frequency stability, where Allan variance needs to be displayed on the HP 5371A's CRT as a fraction of clock frequency (f_c), enter f_c as the normalize factor under the MATH hardkey.

Analysis 1 -- Fractional Frequency Stability

For every $\tau < = 8s$, you can view the Allan variance, a measure of frequency stability, on the instrument's built-in CRT. In fact, the NUMERIC mode displays not only Allan variance $\sigma^2(\tau)$ and root Allan variance $\sigma(\tau)$ but also other useful statistical quantities (see *Figure 10* on page 5). You can view these results as a fraction of the clock frequency. For instance, *Figure 28(a)* shows the root Allan variance of a DS1 clock, with about 1×10^{-8} fractional stability for a τ of 1s.

Analysis 2 - Log $\sigma(\tau)$ vs. τ

By varying s and computing $\sigma(\tau)$, you can also get a log-log plot like *Figure 28(b)* to reveal the sources of random noise. *Figure 25(b)* lists the different types of noise you can determine by examining the slope of the curve, log $\sigma(\tau)$ vs. log. For instance, the example in *Figure 28(b)* shows that phase noise (flicker and white) is the primary cause of instability.

```
Frequency A:                          999.81 m

MATH ─────────────────────────────────────────
                   Channel A          Channel B

Reference             0E+00             0E+00

Statistics    ON               ON

Math:         ON               OFF
   Offset                 0E+0
   Normlz    1.5448000000E+
   Scale     1.0000000000E+0

Limits:       OFF              OFF


Display = [((Result-Ref)/Norm)+Offs]*Scale
```

Figure 27. The MATH functions allow flexibility in terms of what units are displayed. Here "fractional" frequency is desired.

τ Seconds	$\sigma(\tau)$ Hz	$\sigma_y(\tau)$ Fractional
0.01	2.72	1.762E-06
0.1	0.249	1.613E-07
1	0.0157	1.017E-08
8	0.00299	1.937E-09

Frequency: 1,543,997.224111 Hz

Figure 28(a). The measure of Allan variance vs. gate time is a very effective way to characterize frequency stability of wander.
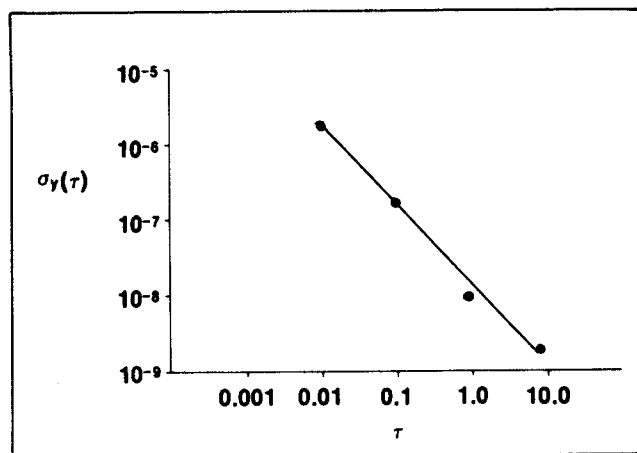

Figure 28(b). A log-log plot reveals the type of instability as being predominantly flicker and white phase noise.

APPENDIX - FOURIER TRANSFORM OF NON-UNIFORMLY SAMPLED DATA

In measuring timing jitter on a stream of either clock or data, the HP 5371A always records phase and timing information at trigger-level crossings nearest to a sampling interval (see *Figure 2*). Because of this measurement characteristic, either timing jitter or random data patterns can invariably cause data to be non-uniformly sampled. To illustrate how to perform an FFT on the HP 5371A's measurement results, which may seem infeasible at first, this appendix explains the following:

1) circumstances under which the effect of non-uniform sampling can be ignored.

2) a recommended algorithm for processing severely non-uniformly sampled data.

When You Can Ignore Non-Uniform Sampling

Data Jitter

Since a stream of data involves non-periodic pulses, the potential severity of non-uniform sampling depends on the number of suppressed zeros that the coding scheme allows from one positive pulse to the next. In general, for non-uniform sampling to be insignificant, the deviation of sampling interval must be small compared to the sampling interval itself:

$$\frac{n}{f_c} \ll T_s$$

n = maximum number of clock cycles between two trigger-level crossings

f_c = clock rate

T_s = Sampling Interval

For instance, in bipolar, return-to-zero coding of DS3 signals (f_c = 44.736 Mb/s), as many as 6 clock cycles can elapse between two positive trigger-level crossings. According to the above formula, in order to ignore non-uniform sampling effects, the sampling interval used must be much greater than 134 ns. When using the HP 5371A for analyzing signals of DS3 or higher data rates, select INTERVAL SAMPLING with intervals of 600 ns or greater and ignore the non-uniformity of the sampled data.

How to Process Non-Uniformly Sampled Data

If the HP 5371A's measurement results are severely non-uniform in time, the following should be done before performing an FFT analysis:

a) Linearly Reconstruct Jitter J(t) - connect the sampled data linearly as shown in *Figure 29(b)*.*

b) Determine a New, Fixed Sampling Interval - Divide the total measurement time by (N-1), where N can be either 512, 1024, 2048 or 4096.

c) Re-sample J(t) - using linear interpolation, re-sample jitter at the new sampling rate as determined in (b). See *Figure 29(b)* and *(c)*.

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* A more elegant technique is to apply a cubic spline fit to the collected data, and subsequently re-sample, but this takes more processing time.