Measuring Phase Spectral Density Of Synthesized Signal Sources Exhibiting f_0 and f^{-1} Noise Characteristics With The 5390A Frequency Stability Analyzer

APPLICATION NOTE 225

HEWLETT
PACKARD

Measuring Phase Spectral Density Of Synthesized Signal Sources Exhibiting f_0 and f^{-1} Noise Characteristics With The 5390A Frequency Stability Analyzer

Introduction

The 5390A is capable of measuring Phase Spectral Density of various signal sources over the offset frequency range of 0.01 Hz to 10 kHz with the primary emphasis in the range below 100 Hz. The method used to make these measurements utilizes time domain measurements using a frequency counter sampling in a specific manner and relating the variance of a series of measurements to the phase spectral density. The measurements made in the time domain result in a digital filter transfer function which has the characteristics of very narrow bandwidths; thus, measurements can be made very close to the carrier, which most other techniques are unable to cover.

As with all digital sampling techniques, the digital filter has harmonic responses which are usually undesirable. In the majority of cases of measurement of close-in phase noise, this is not a problem with the 5390A due to the characteristics of the phase noise spectrum in this close-in region. However, when measuring certain types of oscillators at the upper range of the 5390A, certain errors can occur. This application note discusses the causes and circumstances under which these errors may occur and some methods for reducing them to acceptable levels.

Figure 1(a) is a generalized noise model of an oscillator (from NBS Technical Note 679, *"Frequency Domain Stability Measurements: A Tutorial Introduction,"* which provides an excellent overview of the subject). This model shows five separate noise processes at work, each of which may predominate over a region of the offset frequency spectrum. A high-quality crystal oscillator will usually exhibit phase noise characteristics similar to this. *Figure 1(b)* shows phase noise characteristics of a synthesized signal—in this case, the output of the HP 8660C Synthesized Signal Generator—in the region 10 Hz to 10 kHz, as measured with a wave analyzer. Note that this spectrum exhibits a predominance of flicker phase noise (f^{-1} dependence of frequency), due to the phase-lock loop and frequency multiplication circuitry used to generate the signal. The spectrum also contains a number of spurious outputs, some of which are related to the line frequency (e.g., at 120 Hz) and some related, again, to the synthesis technique.

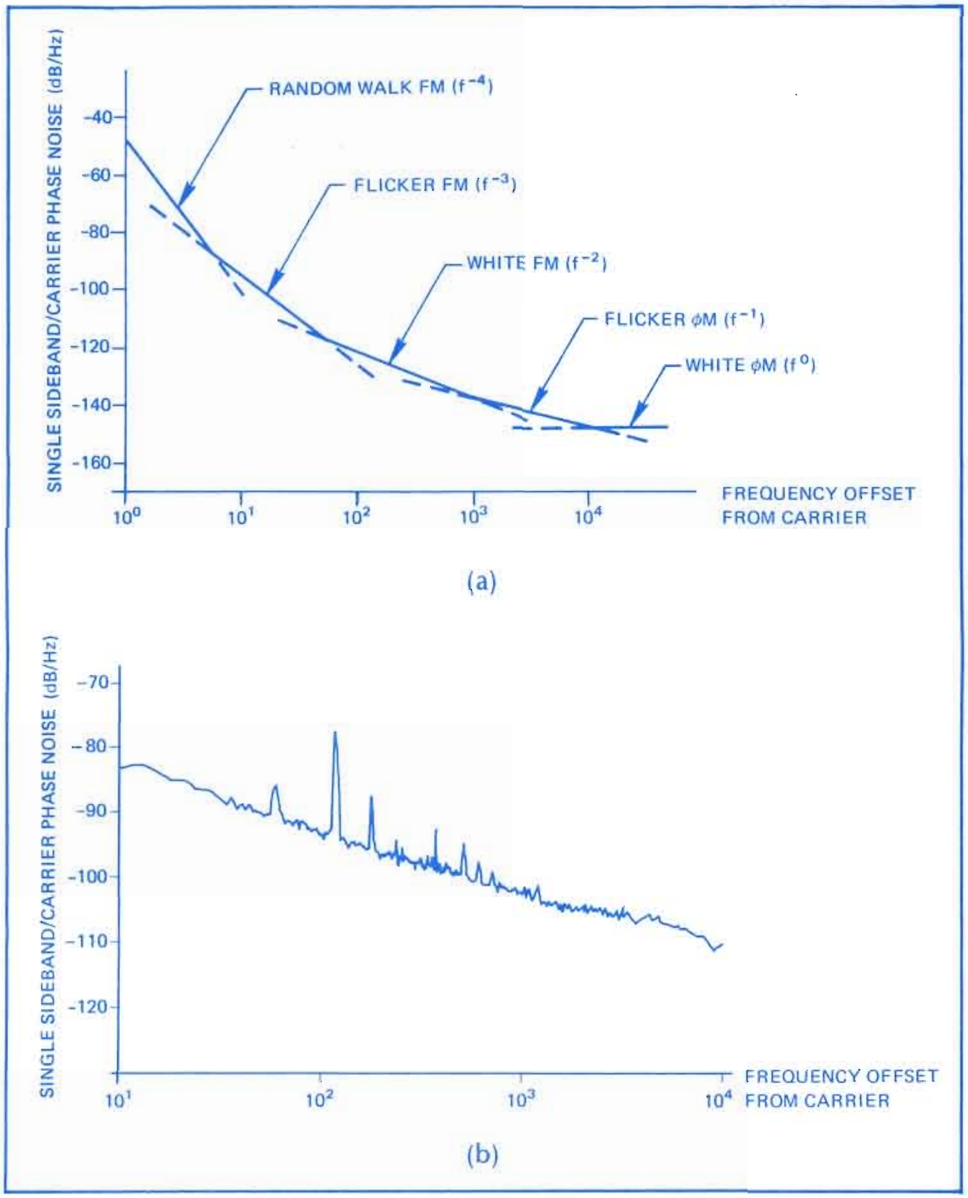

Figure 1. Comparison of phase spectra of (a) idealized oscillator model and (b) HP 8660C (with Option 004) Synthesized Signal Generator in the region 10 Hz—10 kHz. The output frequency of the 8660C is 500 MHz. This measurement was performed using an automated wave analyzer with 1 Hz bandwidth.

Figure 2b shows the characteristic filter function of the 5390A, which is produced by its digital sampling technique. This function includes a fundamental response at the desired offset frequency f on either sideband of the down-converted carrier (beat frequency is denoted ν_b), and additional responses present at all odd harmonics of the fundamental except those divisible by 3. The harmonics continue to infinity on the upper sideband (a low-pass filter on ν_b is included in the system to suppress the higher-order harmonics). The lower sideband harmonics effectively stop at zero, since the diagram is symmetric about the origin. This filter function contrasts with that of a wave analyzer (a), which has a single filter at the offset frequency.

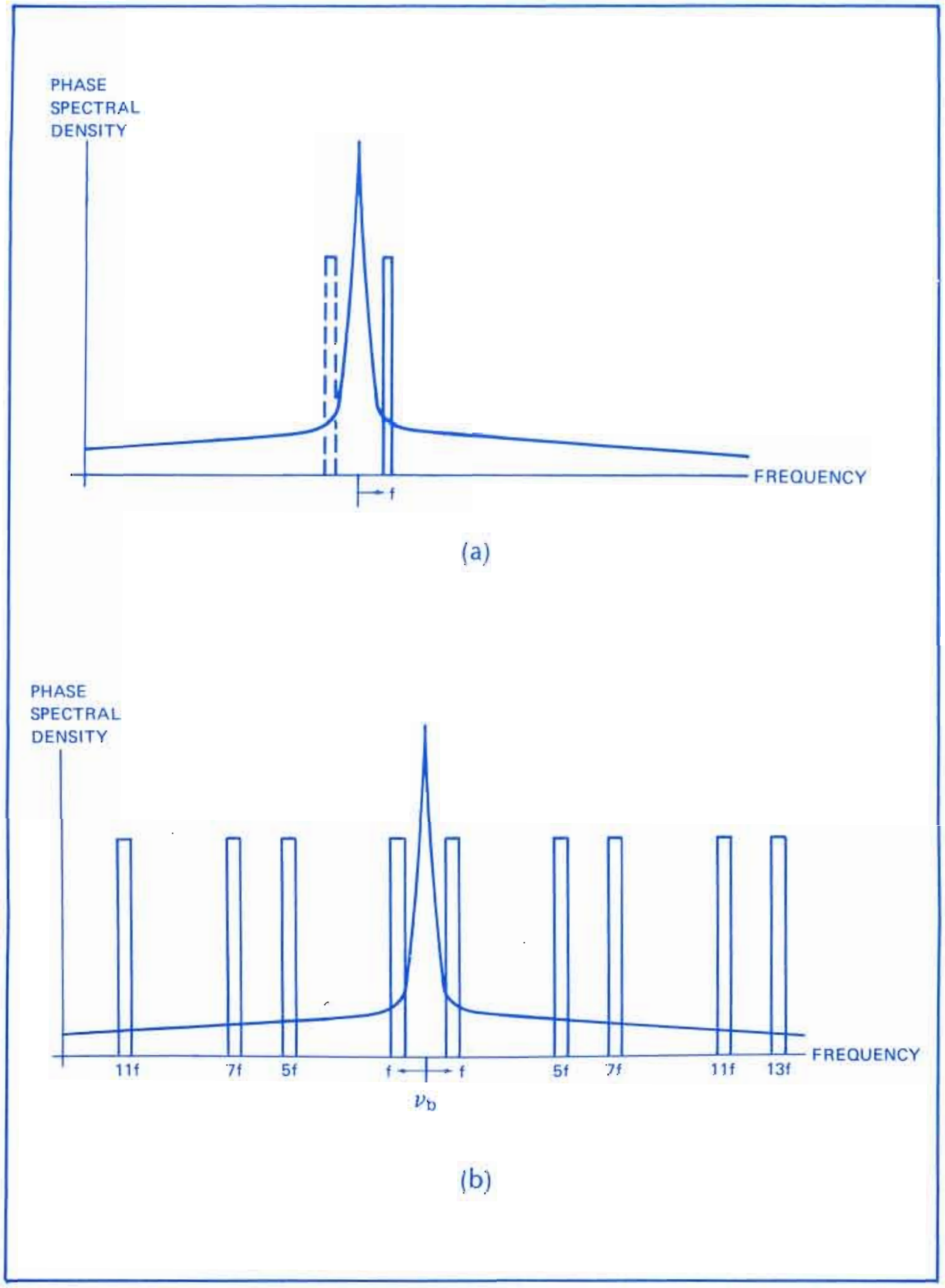

Figure 2. Comparison of filter functions of wave analyzer (a) and 5390A (b). Wave analyzer filter shown in dashed lines is alias filter which occurs only in zero beat techniques, not in IF technique. νb denotes beat frequency. Frequency axis has a linear scale.

Although this filter function appears unweildy, it has a number of unique advantages: It can be constructed with an arbitrarily narrow bandwidth; it can be located arbitrarily close to the carrier; and it contains a true notch at the carrier.

When the 5390A is used to measure phase noise of an oscillator in the f^{-4} to f^{-2} region of *Figure 1(a),* the filter harmonics have negligible effect, since the phase spectral density is falling off so fast away from the carrier. The degree to which this can be a problem can be seen by computing the error contribution of the harmonic responses for the various noise processes. This is done in Appendix B. For the noise processes f^{-4}, f^{-3}, and f^{-2}, the cumulative error is <0.5 dB and can generally be neglected. Since the majority of oscillators exhibit this type of behavior close into the carrier where the 5390A makes its major contribution, no additional steps need be taken.

A worst-case 5390A measurement of the phase noise of *Figure 1(b)* results in the graph of *Figure 3*. The 5390A data in this case is about 5 dB higher than the wave analyzer data. The cause of this discrepancy is the harmonic responses of the digital filter to the f^{-1} noise characteristic.

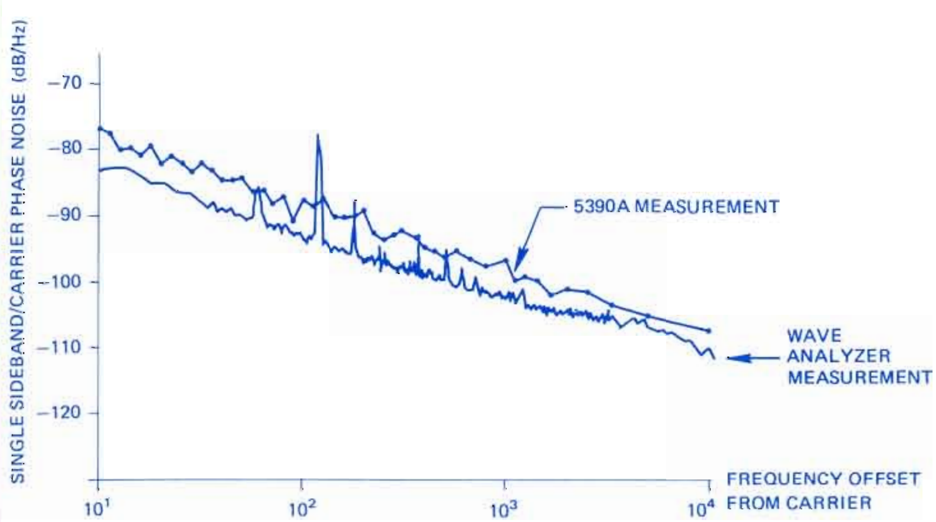

Figure 3. 5390A Frequency Stability Analyzer "wide open" measurement of spectrum of 8660C output at 500 MHz (same as Figure 1b). The beat frequency (νb) for the 5390A measurement is 60 kHz.

The filter harmonics do impact measurements on some synthesized signals as in *Figure 1(b)*, however, for two reasons: First, a filter harmonic may fall on a spurious output; and second, the relatively flat phase-noise spectrum (dominant f^{-1} frequency dependence) over a wide region causes the accumulated power in the filter harmonics to contribute a significant error. As an example of this second effect, let us calculate the error in the measurement of the 8660C at an offset frequency f = 5 kHz, assuming a 60 kHz beat frequency, νb. This situation is illustrated in *Figure 4.* Unwanted filter harmonics occur at 5f, 7f, and 11f on the lower sideband and at 5f and 7f on the upper sideband. Higher-order harmonics are suppressed by the 5390A's 100 kHz low-pass filter. Terms which contribute to this measurement are:

Offset Frequency	**Power Relative to Carrier**
f: 5 kHz	−109 dB
5f: 25 kHz	−113 dB
7f: 35 kHz	−115 dB
11f: 55 kHz	−117 dB
TOTAL	−107 dB

Thus we find that filter harmonics would be expected to bias this point about 2 dB high. The presence of a spur within any of the unwanted filter harmonics would add to this error.

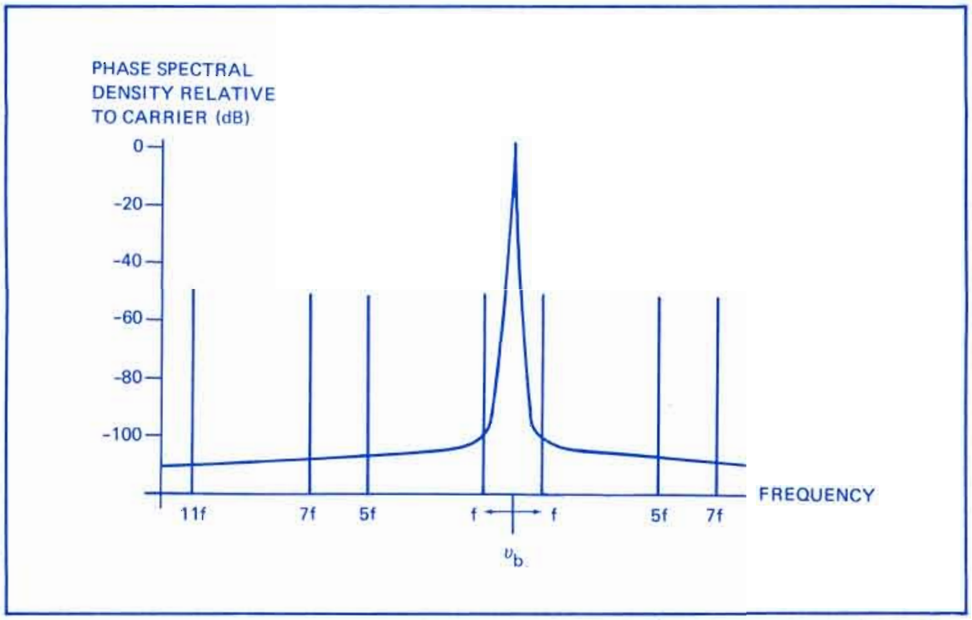

Figure 4. Example of effect of filter harmonics on 5390A's measurement of 8660C.

Methods to Reduce the Effect of Filter Harmonics

1. **Use a low-pass filter with cut-off just above ν_b, and keep ν_b as low as possible.**

   Figure 5 illustrates the effect of the low-pass filter, which is to suppress filter harmonics in the upper sideband. An externally-supplied filter may be used, or the 5390A's set of internal filters may be used:

   Low-pass Filters Incorporated into 5390A

25 Hz	1.6 kHz
100 Hz	8.3 kHz
400 Hz	25 kHz
	100 kHz

   These filters fall off at 6 dB per octave above the cut-off frequency except for the 100 kHz filter, which falls off at 42 dB per octave.

   A low ν_b has the effect of reducing the total number of filter harmonics within the filter's passband. *Figure 6* shows the effect of reducing ν_b from 2 kHz to 1 kHz when measuring phase noise at an offset of 167 Hz; in this case, the number of unwanted filter harmonics has been reduced from three to one. In a similar fashion, a combination of low ν_b and a low-pass filter can greatly reduce the influence of filter harmonics for most measurements.

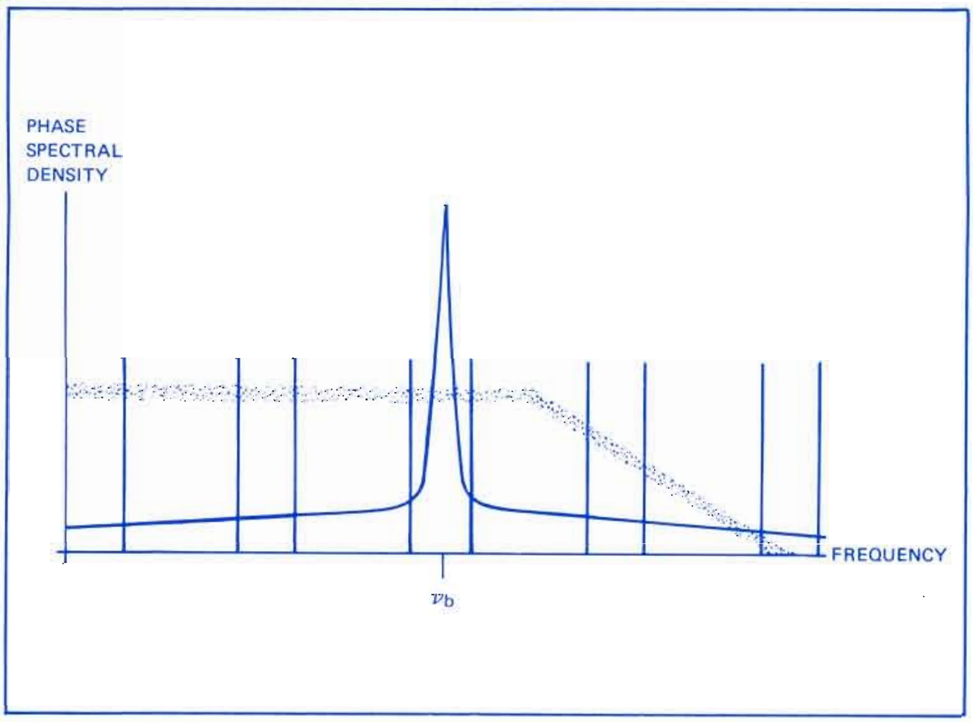

Figure 5. Using a low-pass filter on the IF of the 5390A system to reduce effects of filter harmonics.

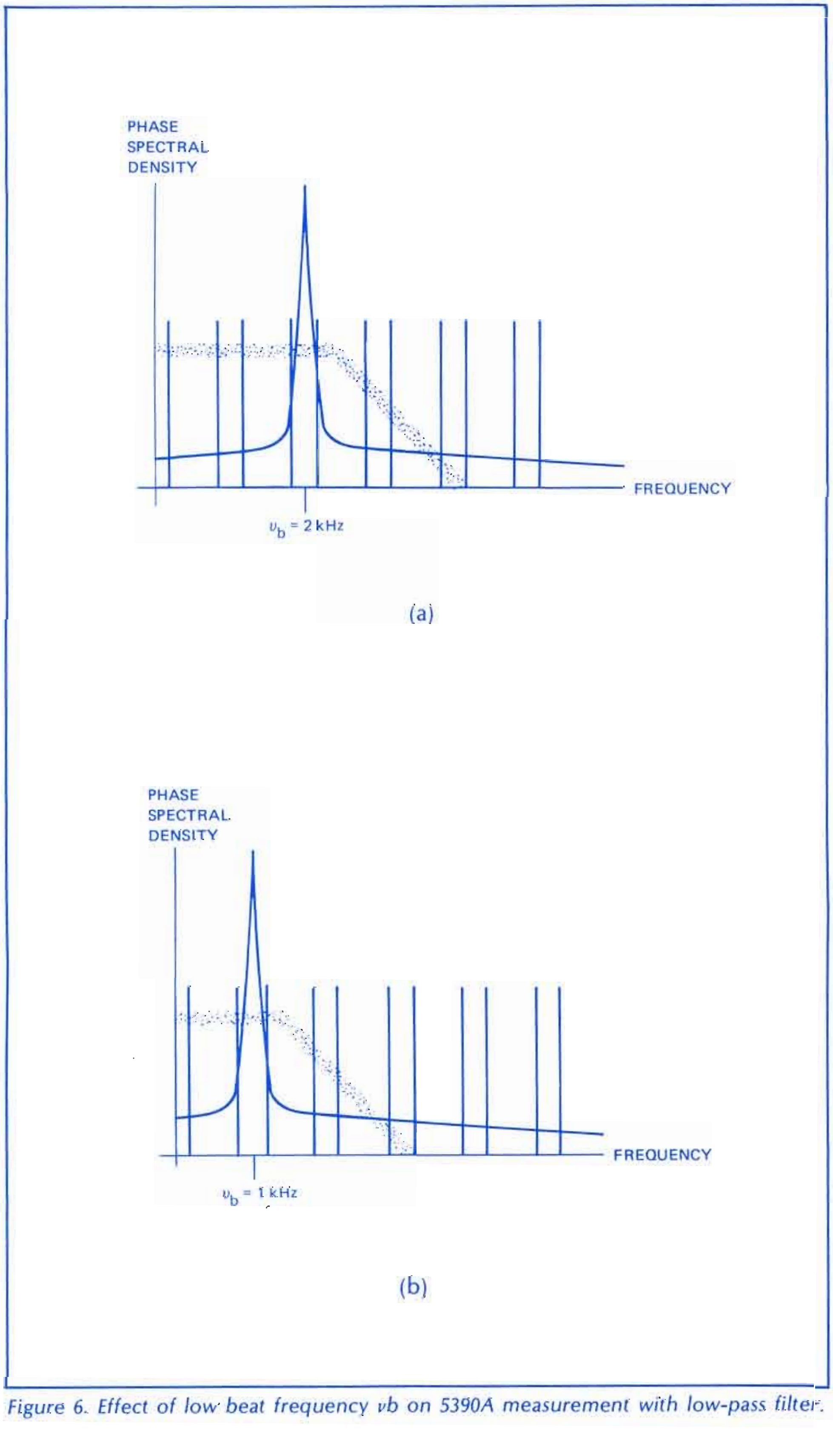

Figure 6. Effect of low beat frequency νb on 5390A measurement with low-pass filter.

2. For optimal results, center a bandpass filter about νb.

This technique is illustrated in *Figure 7.* A carefully selected filter can, of course, eliminate the effects of all filter harmonics. A bandpass filter can be supplied externally or constructed using an external high/low-pass filter and/or the 5390A's low-pass filters. Details of implementing this technique are given in Appendix B.

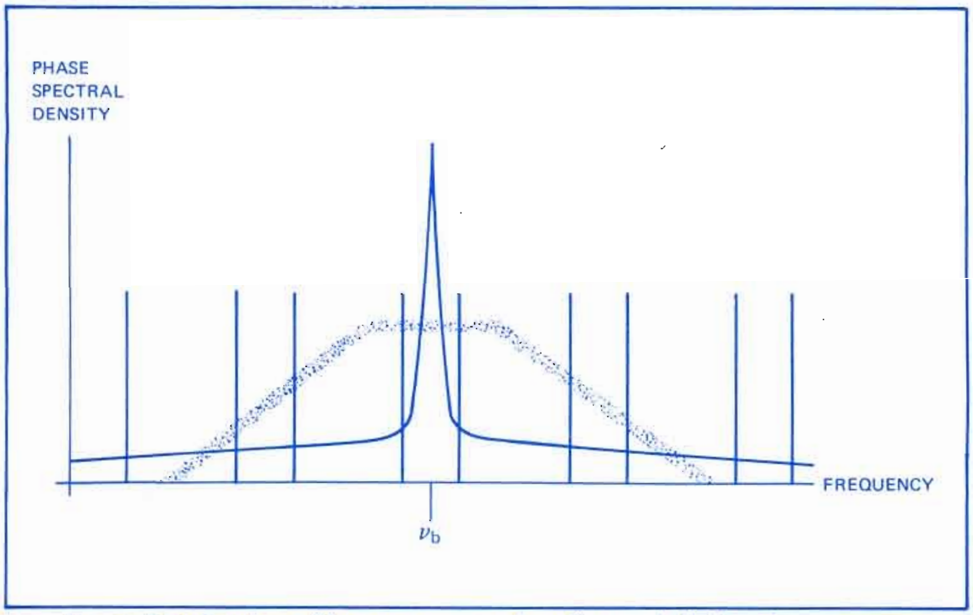

Figure 7. A bandpass filter can negate the effects of all filter harmonics.

Figure 8 shows the results of a 5390A measurement of phase noise of the 8660C under the same conditions as *Figure 3*, except that an external bandpass filter has been applied to ν_b. The additional filtering has eliminated the high bias of results found in *Figure 3*.

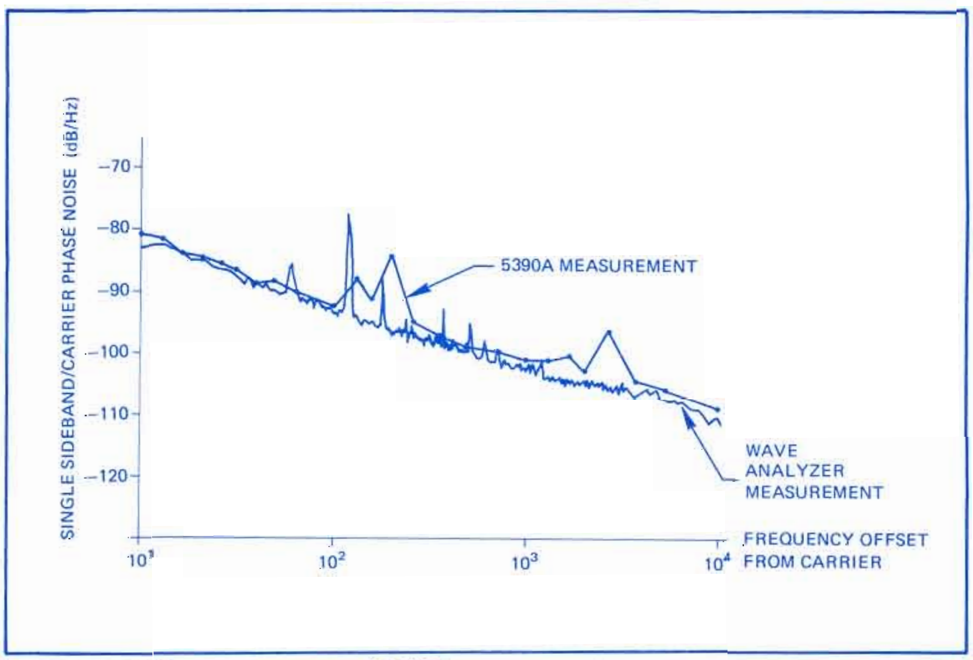

Figure 8. 5390A measurement of 8660C spectrum using appropriate bandpass filters.

The remaining discrepancy between the 5390A data and wave analyzer data in *Figure 8* is due to the location and magnitude of spurious outputs. Differences in location are due to the fact that data points for these two instruments were not taken at exactly the same offset frequencies and at different bandwidths.

If the sources under test contained no spurious responses or bright lines, the plots from the 5390A and wave analyzer would be identical within the measurement resolution. In both measurement methods the phase noise is normalized to a per Hertz basis when plotted. This assumes that the measured noise is evenly distributed across the bandwidth of the measuring filter. The measured value is then divided by the equivalent filter bandwidth in order to normalize it to a per Hertz value.

When spurious responses or bright lines are present within the filter bandwidth energy is not evenly distributed across the measuring filter bandwidth, and a peak rather than an average value is desired. Hence, to gain close correlation of spurious response or bright line measurements with both 5390A and wave analyzer techniques, the values plotted should be "un-normalized" by the measuring filter bandwidth. With the 5390A this is done by adding to the plotted value 10 times the log of the measurement filters' bandwidths. Wave analyzer values should be correct in a similar manner.

APPENDIX A

ERROR ANALYSIS OF DIGITAL FILTER HARMONIC RESPONSE FOR VARIOUS NOISE PROCESSES

The error due to the harmonic responses of the digital filter can be expressed as follows:

$$\text{error} = 10 \log \left[1 + \left(\frac{f}{f_o}\right)^{\beta} \left| H(f) \right|^{2} \right] \qquad \text{(A1)}$$

where $\frac{f}{f_o}$ represent the harmonic number

β is the phase power law coefficient $(0 < \beta < -4)$

H(f) is the transfer function of any signal conditioning of the spectra prior to sampling.

Eq. A1 assumes that the error over the frequency range of interest is due to a signal noise process, otherwise the expression would have to be expanded to evaluate the contribution of each noise process over the range of interest. In practice, one noise process usually predominates and the contribution of the others is negligible.

Table A1 evaluates Eq. A1 for the cumulative error of N harmonic responses. That is

$\left|H(f)\right|^2 = 1$ for $f_o \leq f \leq Nf_o$ and $\left|H(f)\right|^2 = 0$ elsewhere.

TABLE A1

ERROR DUE TO FILTER HARMONIC RESPONSES

Harmonic Number	Cumulative Error (dB)				
	Noise Process				
	f0	f−1	f−2	f−3	f−4
1	0.0	0.0	0.00	0.000	0.000
5	3.0	0.7	0.17	0.035	0.007
7	4.7	1.2	0.25	0.047	0.009
11	6.0	1.5	0.28	0.050	0.009
13	6.9	1.7	0.31	0.052	0.009
17	7.7	1.9	0.32	0.053	0.009
19	8.4	2.1	0.33	0.054	0.009
23	9.0	2.2	0.34	0.054	0.009
25	9.5	2.3	0.35	0.054	0.009
29	10.0	2.4	0.35	0.055	0.009
31	10.4	2.4	0.36	0.055	0.009
35	10.7	2.5	0.36	0.055	0.009
37	11.1	2.6	0.36	0.055	0.009
41	11.4	2.6	0.36	0.055	0.009
43	11.7	2.7	0.37	0.055	0.009
47	12.0	2.7	0.37	0.055	0.009
49	12.3	2.8	0.37	0.055	0.009
53	12.5	2.8	0.37	0.055	0.009
55	12.7	2.9	0.37	0.055	0.009
59	13.0	2.9	0.37	0.055	0.009
61	13.2	2.9	0.38	0.055	0.009
65	13.4	3.0	0.38	0.055	0.009
67	13.6	3.0	0.38	0.055	0.009
71	13.8	3.0	0.38	0.055	0.009
73	13.9	3.1	0.38	0.055	0.009
77	14.1	3.1	0.38	0.055	0.009
79	14.3	3.1	0.38	0.055	0.009
83	14.4	3.1	0.38	0.055	0.009
85	14.6	3.2	0.38	0.055	0.009
89	14.7	3.2	0.38	0.055	0.009
91	14.9	3.2	0.38	0.055	0.009
95	15.0	3.2	0.38	0.055	0.009
97	15.1	3.3	0.38	0.055	0.009
101	15.3	3.3	0.38	0.055	0.009
125	16.2	3.4	0.39	0.055	0.009
151	17.1	3.6	0.39	0.055	0.009
175	17.5	3.7	0.39	0.055	0.009
203	18.3	3.7	0.39	0.055	0.009
301	20.0	3.9	0.40	0.055	0.009
401	21.3	4.2	0.40	0.055	0.009
503	22.3	4.3	0.40	0.055	0.009
751	24.0	4.5	0.40	0.055	0.009
997	25.2	4.6	0.40	0.055	0.009

APPENDIX B

SEMI-AUTOMATIC IMPLEMENTATION OF BANDPASS FILTERING USING THE 5390A

Introduction

Implementation of the bandpass filtering techniques discussed in this application note can be accomplished by incorporating an external variable bandpass filter in the amplifier chain of the 10830A Mixer IF Amplifier. The 10830A provides an intermediate output of its amplifier chain after the first stage and an input back into the remaining stages of the amplifier allowing the insertion of a filter. By taking the output after the first stage which provides 50 dB (X300) of gain, the signal can be applied to an active high/low filter without severely raising the noise floor of the system. Typically active bandpass filters are implemented as a high pass/low pass cascade configuration. The result signal processing block diagram is shown in Figure B1.

To facilitate the use of the bandpass filter in the measurement process, it is convenient to make some modifications to the phase noise application program supplied with the 5390A. The modifications rearrange the measurement sequence, compute the high pass and low pass cut-off frequencies, prompt the operator to set the filter to these values, and optionally allow printing of intermediate results so that the progress of the measurement can be monitored. All other aspects of the program operation and measurement remain the same.

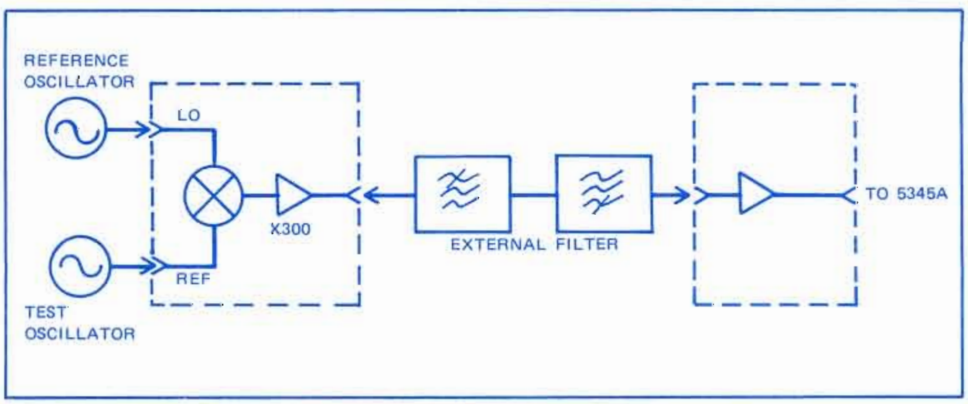

Figure B1. Signal Processing Block Diagram with Bandpass Filtering.

Equipment Set-Up

The only set-up change required to the system configuration is the interconnection of the external filter to the 10830A. This is accomplished by removing the short BNC coax cable (W15) on the rear panel of the 10830A from the two BNC labeled EXTERNAL FILTER OUT and EXTERNAL FILTER IN. The external filter is then connected as shown in Figure B2.

The experimental data shown in this application note was taken using a Rockland Model 852 Dual HI/LOW Filter with the first stage filter set to HI PASS, the second stage set to LO PASS, and both stages set to 0 BD GAIN and FLAT AMPL.

Program Modification

The application program modifications can be performed by the following procedure. It is recommended that the operator be familiar with the basic editing procedures as discussed in the 9825A Calculator Operating & Programming Manual (HP P/N 09825-90000), Chapter 4.

1. Load and Run the phase noise program (pnamh) according to normal 5390A operating procedures.

2. Press Start Measurement key (F5).

3. When the display returns with the first question, press the STOP key.

4. Make the changes and additions per Table B1. Note: the correctness of each change and addition can be verified by listing just those lines shown and comparing the check sums (the number at the end of the listing preceded by an asterisk). If the numbers are not the same, look for differences between the listing and Table B2.

5. After all changes and additions have been entered, list lines 30 to the end and verify the check sum with that of Table B2.

6. Record the updated version of the measurement segment by typing

   ```
   trkØ; rcf z[4] + 4, 30
   ```

   and then pressing EXECUTE (first make sure the tape cartridge record protect tab is positioned in the direction of the arrow (record enabled)).

Note

If the original version of the program is also to be used, first make a duplicate copy of the system software cartridge (only the phase noise program need be copied, if desired), and then perform the edits and record them on the duplicate copy.

Program Operation

The program is operated in the normal manner with the following exceptions:

1. The sweeps are performed all at one frequency at a time, then repeated at the next frequency.

2. The program computes the optimum beat frequency and prompts the user to set it to this value. It then measures the actual beat frequency and computes the measurable frequencies and filter frequencies.

3. Prior to starting each new frequency, the program will prompt the operator to set the bandpass filter high and low pass cut-off frequencies. The display will show

   SET HP = d.ddEdd SET LP = d.ddEdd

   Set the high pass and low pass controls to the values indicated (in Hz) and press CONTINUE.

4. The system will make measurements at this frequency for the number of sweeps indicated, and then pause as in 2. above, for each frequency requested.

5. No print out will occur until a complete line of frequencies has been measured. If it is desired to monitor progress, type sfg 8 EXECUTE, and the intermediate results will be printed on the calculator's internal printer.

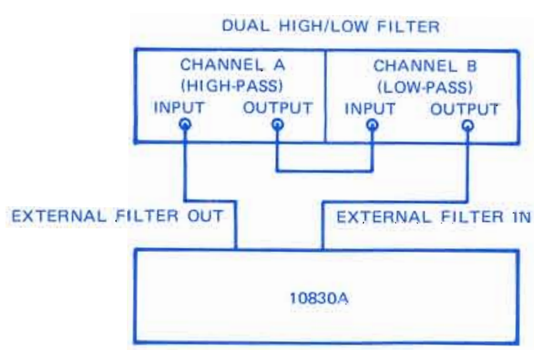

Figure B2. Semi-Automatic Bandpass Filtering Equipment Set-up.

TABLE B1

PROGRAM EDITS FOR MANUAL B.P. FILTER

Note: these edits must be executed in sequence.

Replace line 30 with

```
30: "MEAS & REC PARAM w/man.filter cp.771111.1506":
*15692
```

Replace line 40 with

```
40: fmt /,15x,"*** P H A S E  N O I S E  A N A L Y S I S ***";wrt "prtr"
*10316
```

Fetch line 41, add insert the following

```
41: fmt /,24x,"-- MANUAL EXTERNAL  FILTER --",3/;wrt "prtr"
*13948
```

Fetch line 46 and insert

```
46: 0→I[2];1→J
47: "nf":cll 'bb';dsp "Set Beat Frequency to",6*O[J];stp
*25300
```

Replace line 49 with

```
49: fmt 2/,3x,"IF FREQUENCY",f9.1," HZ";wrt "prtr",F
*23134
```

Replace line 52 with

```
52: I[2]→1;J→I[1];for J=J to O
*16028
```

Fetch line 57 and insert

```
57: if F[I]<F[I[1]]/5.05;I-1→M→I[2];gto +3
*18762
```

Replace line 59 with

```
59: I→M→I[2]
*27084
```

Replace line 62 with

```
62: if F+1.1F[I[1]]<H[1];gto +2
*28953
```

Fetch line 70 and delete
Fetch line 71 and delete

Fetch line 83 and insert

```
83: fmt "HPF",z;wrt "prtr";fmt f8.0,z
84: for I=I[1] to I[2];wrt "prtr",'lf'(I);next I
85: fmt "     HZ";wrt "prtr"
86: fmt "LPF",z;wrt "prtr";fmt f8.0,z
87: for I=I[1] to I[2];wrt "prtr",'hf'(I);next I
88: fmt "     HZ";wrt "prtr"
*16766
```

Replace line 94 with

```
94: for I=I[1] to I[2]
*9170
```

Fetch line 95 and delete
Fetch line 95 and insert

```
95: if not flg8;gto +4
96: spc
97: fxd 1;prt "FREQ(HZ)→ ",F[I],"----------------"
98: prt "SWEEP        DBC "
99: 'lf'(I)→r1;'hf'(I)→r2
100: if I#1;if 'hf'(I-1)=r2;gto +2
101: flt 2;cll 'bt';dsp "SET HP=",r1," LP=",r2;stp
*5518
```

Replace 104 with

```
104: for A=1 to A[2]
*8506
```

Fetch 133 and insert

```
133: if not flg8;gto +2
*9454
```

Replace lines 134 and 135 with

```
134: fmt f4.0,3x,f8.0;wrt 16,A,10log(S[A,I])
135: next A
*5585
```

Replace line 137 with

```
137: next I
*31652
```

Fetch line 138 and insert

```
138: for A=1 to A[2]
139: fmt /,f5.0,z;wrt "prtr",A
140: for I=I[1] to I[2]
141: fmt f8.1,z;wrt "prtr",10log(S[A,I])
142: next I
143: next A
144: wtb "prtr",13
*12014
```

Fetch line 173 and replace with 173:
Fetch line 176 and insert

```
176: if J<0;gto "nf"
*15923
```

Fetch 183 and insert

```
183: "lf":ret drnd(prnd(.9364(F-F[pl]),0),3)
184: "hf":ret drnd(prnd(1.068(F+F[pl]),0),3)
*20240
```

TABLE B2

COMPLETE LISTING OF "MEAS" Segment after Edits

```
30: "MEAS & REC PARAM w/man.filter cp.771111.1506":
31:
32: "me":jmp Z[2]
33: gto "meas"
34: gto "rpad"
35: gto "rpt"
36:
37: "meas":
38: ent "ENTER MEASUREMENT DESCRIPTION",M$
39: wtb "prtr",12
40: fmt /,15x,"*** P H A S E   N O I S E   A N A L Y S I S ***";wrt "prtr"
41: fmt /,24x,"-- MANUAL EXTERNAL  FILTER --",3/;wrt "prtr"
42: wrt "prtr.1"
43: fmt 2/,"MEASUREMENT DESCRIPTION:",2/,3x,c;wrt "prtr",M$
44: cll 'rapt';A$+T$
45: fmt "MEASUREMENT PARAMETERS:",/;wrt "prtr"
46: 0+I[2];1+J
47: "nf":cll 'bb';dsp "Set beat Frequency to",6*U[J];stp
48: cll 'mf'(F)
49: fmt 2/,3x,"IF FREQUENCY",f9.1," HZ";wrt "prtr",F
50:
51: "COMPUTE OFFSET FREQS":
52: I[2]+1;J+I[1];for J=J to 6
53: if U[J]>F/6;F/6+F[I+I+1];gto +2
54: F/(6*prnd(F/(6*U[J]),0))+F[I+I+1]
55: if (prnd(U[J]/B[J],0)+N[I])<1;cll 'pe'(11)
56: if I#1;if F[I]=F[I-1] or F[I]>6*F;I-1+I
57: if F[I]<F[I[1]]/5.05;I-1+I+I[2];gto +3
58: next J
59: I+ .+I[2]
60:
61: for I=1 to 7
62: if F+max(F[*])<H[I];gto +2
63: next I
64: fxd 0;dsp "set IF BANDWIDTH to",H[I]+H;gsb "bb"
65: ent "",H
66: fmt 3x,"IF BANDWIDTH ",f7.0," HZ";wrt "prtr",H
67: fmt 3x,"K=",f4.0;wrt "prtr",K[2]
68: fmt 3x,"CORRECTION COEFFICIENT",f4.0;wrt "prtr",C
69:
70: "rpt":
71: fmt 2/,10x,"SSB/CARRIER PHASE NOISE (DB/HZ)";wrt "prtr"
72: fmt "FREQ+",z;wrt "prtr"
73: fmt f8.2,z;for I=I[1] to I[2];wrt "prtr",F[I];next I
74: fmt "  HZ";wrt "prtr"
75: fmt "BW  +",z;wrt "prtr"
76: fmt f8.2,z
77: for I=I[1] to I[2];wrt "prtr",F[I]/N[I];next I
78: fmt "  HZ";wrt "prtr"
79: fmt "FLOOR",z;wrt "prtr"
80: fmt f8.1,z
81: for I=I[1] to I[2];wrt "prtr",-173+20log(F)-10log(F[I]);next I
82: fmt "  DB";wrt "prtr"
83: fmt "HPF",z;wrt "prtr";fmt f8.0,z
84: for I=I[1] to I[2];wrt "prtr",'lf'(I);next I
85: fmt "     HZ";wrt "prtr"
86: fmt "LPF",z;wrt "prtr";fmt f8.0,z
87: for I=I[1] to I[2];wrt "prtr",'hf'(I);next I
88: fmt "     HZ";wrt "prtr"
89: fmt "SWEEP",z;wrt "prtr"
90:
91: "SET-UP FOR SWEEPS":
92: cmd "cntr","I2E;G5E1<E811";cmd "msp","G1G3G4D2F1H";wait 50
93: rds("msp")+r1
94: for I=I[1] to I[2]
95: if not flg8;gto +4
96: spc
97: fxd 1;prt "FREQ(HZ)+ ",F[I],"----------------"
98: prt "SWEEP          DBC "
99: 'lf'(I)+r1;'hf'(I)+r2
100: if I#1;if 'hf'(I-1)=r2;gto +2
101: flt 2;cll 'bb';dsp "SET HP=",r1," LP=",r2;stp
```

```
102:
103: "SET-UP FOR NEW FREQ":
104: for A=1 to A[2]
105: 1e6/(3*F[I])→G
106: int(log(G*500))+2→D
107: if G<=100;fmt "D2M",f.0,"EOK";wrt "msp,pp",G;gto +2
108: fmt "D2M",e.2,"K";wrt "msp,pp",G/100
109: fmt "T",f.0,"N",f.0,"RI";wrt "msp,pp",D,2*N[I]
110:
111: "MEAS SIGMA":
112: 0→M[2]→M[3]
113: for K=1 to K[2]
114: 0→M[1]
115: fmt z;red "msd";fmt
116: for N=1 to N[I]
117: red "bus",T,E;(E→Y)/T→X
118: red "bus",T,E
119: if E=Y or E=Y+1 or E=Y-1;gto +2
120: if flg10;if N#1;flt 5;prt "S=",A,"F=",F[I],"K=",K,"N=",N,"dF",X-E/T;spc
121: X-E/T+M[1]→M[1]
122: next N
123: band(rds("msp"),125)→r1;if r1#96;cll 'pe'(5);dsp dtorl
124: if K#K[2];fmt "RI",z;wrt "msp,pp"
125: (M[1]*5e8)↑2+M[2]→M[2]
126: M[1]*5e8+M[3]→M[3]
127: next K
128:
129: "COMPUTE VARIANCE & S":
130: (M[2]-M[3]↑2/K[2])/(K[2]-1)→Z
131: Z/(5.45*F[I]↑3*N[I]*C)→S[A,I]
132: if (tn↑((-176+20log(F)-10log(F[I]))/10)→r1)>S[A,I]:r1→S[A,I]
133: if not flg8;gto +2
134: fmt f4.0,3x,f8.0;wrt 16,A,10log(S[A,I])
135: next A
136:
137: next I
138: for A=1 to A[2]
139: fmt /,f5.0,z;wrt "prtr",A
140: for I=I[1] to I[2]
141: fmt f8.1,z;wrt "prtr",10log(S[A,I])
142: next I
143: next A
144: wtb "prtr",13
145: "COMPUTE AVE,SIGMA,MIN & MAX":
146: fmt /,4x,z;wrt "prtr"
147: fmt "  ------",z;for I=I[1] to I[2];wrt "prtr";next I
148: for I=I[1] to I[2]
149: 0→r1→r2→r5;1e99→r3;-1e99→r4
150: for A=1 to A[2]
151: S[A,I]+r1→r1
152: (10log(S[A,I]))↑2+r2→r2
153: min(S[A,I],r3)→r3
154: max(S[A,I],r4)→r4
155: 10log(S[A,I])+r5→r5
156: next A
157: r1/A[2]→S[A[2]+1,I]
158: if A[2]=1;gto +4
159: √abs((r2-r5↑2/A[2])/(A[2]-1))→S[A[2]+2,I]
160: r3→S[A[2]+3,I]
161: r4→S[A[2]+4,I]
162: next I
163: "print":
164: fmt /,"AVE=",z;wrt "prtr"
165: fmt f8.1,z;for I=I[1] to I[2];wrt "prtr",10log(S[A[2]+1,I]);next I
166: if A[2]=1;gto +7
167: fmt /,"SIG=",z;wrt "prtr"
168: fmt f8.1,z;for I=I[1] to I[2];wrt "prtr",S[A[2]+2,I];next I
169: fmt /,"MIN=",z;wrt "prtr"
170: fmt f8.1,z;for I=I[1] to I[2];wrt "prtr",10log(S[A[2]+3,I]);next I
171: fmt /,"MAX=",z;wrt "prtr"
172: fmt f8.1,z;for I=I[1] to I[2];wrt "prtr",10log(S[A[2]+4,I]);next I
173:
174:
175: cll 'mf'(r1);fmt 2/,"IF DRIFT",f12.6," hz";wrt "prtr",E-r1
176: if J<C;gto "nf"
177: cmd "cntr","E142I1"
178: if flg9;fmt c,z;wrt "prtr","END ";cll 'rapt'
```

```
179: gto "ex"
180:
181: "mf":cmd "cntr","I2G?E1<E:E8I1";fmt ;red "cntr",p1;red "bus",p1;ret
182: "bb":beep;wait 200;beep;ret
183: "lf":ret drnd(prnd(.9364(F-F[p1]),0),3)
184: "hf":ret drnd(prnd(1.068(F+F[p1]),0),3)
185:
186: "rapt":
187: fmt ;red "clk";"  "→A$
188: wti 0,7;rdi 4→D;wait 100;if iof7;gto +2
189: cfg 9;ent "ent date/time MM,DD,hh:mm:ss",A$[3,16];gto +2
190: sfg 9;red "clk",A$;if A$[1,1]="?";wrt "prtr","CLOCK ERROR"
191: Y$&A$[1,16]→A$
192: fmt 3x,"DATE ",c2,"/",c2,"/",c2,"  TIME  ",c,2/
193: wrt "prtr",A$[1,2],A$[5,6],A$[8,9],A$[11,18];ret
194:
195: "rpad":
196: ent "enter record file #",r1
197: if 'cf'(r1);gto -1
198: L[1]→T[1];L[2]→T[2]
199: rcf r1,M$,T$,K[*],A[*],C,X[*],Y[*],C[*],T[*],O
200: if 'cf'(r1+1);gto -4
201: rcf r1+1,F,M,O[*],F[*],B[*],S[*]
202: gto "ex"
203:
204: "cf":
205: fdf p1
206: idf p2,p3,p4,p5,p6
207: if p4≠0;fxd 0;dsp p1,"used.type",p3,"ok to destroy?";gto +2
208: ret 0
209: ent "",A$
210: if A$[1,2]="ye";ret 0
211: if A$[1,2]="no";ret 1
212: cll 'pe'(8);gto -3
*2312
```

Measurement Considerations

In order for the bandpass filter to be effective, it is necessary to set its cutoff frequency with respect to the beat frequency and offset frequencies such that the filter pass and stop band characteristics can provide adequate filtering. Two conditions should be observed: 1) The pass band loss at the offset frequency being measured should be kept to a minimum. This is especially important when corner frequency gets close to the beat frequency. 2) The stop band loss at the harmonic responses of the digital filter should be great enough to make their contribution to the measurement negligible. These two conditions limit how close to the carrier frequency can be measured with a given beat frequency and filter characteristic. The upper offset frequency limit is governed by the same considerations as making the measurement without the bandpass filter, mainly

$$f_{o_{max}} = \frac{\nu_b}{6}$$

To determine the offset frequency operating limits with a given filter characteristic, first choose an acceptable pass band loss. This is the maximum allowable loss at the fundamental response of the digital filter. For example, choose −0.5 dB, which is commensurate with the statistical validity of typical measurements. Next, choose an acceptable error limit due to the filter harmonic responses. Again, for example, select 0.5 dB. From this can be computed the necessary stop band rejection of the filter. If the filter is of a high enough order (>24 dB/oct rolloff), then the error is predominantly due to the 5th harmonic of the digital filter and the required attenuation can be calculated by the following value of the filter's transfer function:

$$\left|H(f_b \pm f_o)\right|^2 = 10 \log \left|\log^{-1}\left(\frac{\epsilon}{10}\right) - 1\right| \text{ dB}$$

where ϵ is the allowable error in dB.

Now choose a filter type and order. For an example, a cascade high and low pass 8-pole butterworth will be used, as such a filter is readily available. For convenience, let the following coefficients be defined:

$$k_1 = \frac{\nu_b + f_o}{f_h} \qquad k_2 = \frac{\nu_b + 5\,f_o}{f_h} \text{ low pass section}$$

$$k_3 = \frac{\nu_b - f_o}{f_l} \qquad k_4 = \frac{\nu_b - 5\,f_o}{f_l} \text{ high pass section}$$

where ν_b = beat frequency
f_o = offset frequency from carrier
f_l = low frequency corner frequency of the high pass section
f_h = high frequency corner frequency of the low pass section

Using the transfer function of the filter, calculate the values of the h_i's using the two values of the transfer function previously selected and computed. For the example, the low pass section will be evaluated and because of symmetry and reciprocity the high pass characteristic will follow.

Thus, for the 8-pole butterworth sections

$$k_1 = \left[\frac{1}{|H_h(\nu_b + f_o)|^2} - 1\right]^{\frac{1}{2n}} = \frac{1}{k_3}$$

$$k_2 = \left[\frac{}{|H_h(\nu_b + 5f_o)|^2} - 1\right]^{\frac{1}{2n}} = \frac{1}{k_2}$$

where n is the filter order (16 for the example).

The criterion is that the corner frequency must lie between the two filter performance limits. This can be expressed as follows:

$$\frac{\nu_b + f_o}{k_1} \leqslant f_n \leqslant \frac{\nu_b + 5f_o}{k_2}$$

and thus, the minimum offset frequency that can be measured is given by

$$f_{omin} = \frac{\frac{k_2}{k_1} - 1}{5 \frac{k_2}{k_1}} \nu_b$$

It is also convenient to define a constant R as

$$R \triangleq \frac{f_{omax}}{f_{omin}}$$

Since $f_{omax} = \frac{\nu_b}{6}$ then R can be computed by

$$R = \frac{1}{6} \frac{\left(5 - \frac{k_2}{k_1}\right)}{\left(\frac{k_2}{k_1} - 1\right)}$$

R can then be used to compute the range of offset frequencies assuming f_b is always set to 6 $f_{o/max}$. For our example filter, R = 4.96 ~ 5. Thus, using this, a

table of measurable frequencies can be generated as shown in Table A3.

TABLE A3.
Measurable Frequencies with 8-pole Butterworth Filters
R = 5

f_b	f_{omax}	f_{omin}	Measureable Frequencies*
60K	10K	2K	10, 5, 3.3, 2.5, 2
6K	1K	200K	1K, 500, 330, 250, 200
600K	100K	20K	100, 50, 33, 25, 20
60K	10K	2K	10, 5, 3.3, 2.5, 2

$^*f_o = \frac{\nu_b}{6\,\iota} \qquad \iota = 1, 2, 3, \ldots$

HEWLETT PACKARD

For more information, call your local HP Sales Office or East (301) 948-6370 • Midwest (312) 255-9800 • South (404) 955-1500 • West (213) 877-1282. Or, write: Hewlett-Packard, 1501 Page Mill Road, Palo Alto, California 94304. In Europe, Post Office Box, CH-1217 Meyrin 2, Geneva, Switzerland. In Japan, Yokogawa-Hewlett-Packard, 1-59-1, Yoyogi, Shibuya-Ku, Tokyo, 151.

02–5952–7461 February 1978 Printed in USA