Spectral Analysis Using a Deep-Memory Oscilloscope Fast Fourier Transform (FFT)

For Use with Infiniium 54830B Series Deep-Memory Oscilloscopes

Application Note 1383-1

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Introduction

Many of today's digital oscilloscopes include a Fast Fourier Transform (FFT) for frequency-domain analysis. This feature is especially valuable for oscilloscope users who have limited or no access to a spectrum analyzer yet occasionally need frequency-domain analysis capability. An integrated oscilloscope FFT provides a cost effective, space saving alternative to a dedicated spectrum analyzer. Though the latter does exhibit better dynamic range and less distortion, the digital oscilloscope offers several compelling benefits.[1]

The Agilent Technologies Infiniium 54800 Series digital oscilloscopes include FFT functions for computing both magnitude and phase. Several useful features assist in spectral analysis. Controls adjust memory depth, sampling rate, vertical scale and horizontal scale of the FFT. Automatic measurements and markers measure spectral peak frequencies and magnitudes as well as deltas between peaks. The Infiniium Help system provides extensive information on FFT theory and application. Several features designed primarily for time-domain analysis are also useful for the FFT. Display traces can be annotated and saved to a file. The oscilloscope configuration can be saved and recalled as a setup file. Functions can be chained together to perform complex tasks, such as computing the average, maximum or minimum of several FFT spectrums. Measurement statistics are available for computing the mean and standard deviation of a measurement over several acquisitions. With all this capability, the oscilloscope FFT provides a very convenient tool for spectral analysis.

The deep-memory Infiniium 54830B family of oscilloscopes enable an increase in the record length of the FFT, which in turn improves the frequency spectrum estimate. Longer record lengths provide finer frequency resolution and better dynamic range. By upgrading the processor speed and improving the efficiency of the FFT algorithm, the Infiniium deep-memory oscilloscopes can perform FFTs on long records very quickly.

Agilent Technologies

FFT Fundamentals

This application note begins with a discussion of FFT fundamentals and highlights those characteristics that are important for understanding FFT-based spectral analysis. It goes on to explain practical considerations for using an oscilloscope FFT for spectral analysis and techniques that can be used to improve dynamic range and accuracy. Finally, it concludes with a specific application that illustrates the benefits of a deep-memory oscilloscope FFT for making high-frequency resolution spectral measurements.

Discrete Fourier Transform

To better understand the limitations of using an oscilloscope FFT for spectral analysis, it is important to understand some fundamental properties of the Discrete Fourier Transform (DFT) and the effects of sampling. A brief review is covered here. The reader is referred to other sources for more detailed information.

The DFT represents discrete samples of the continuous Fourier transform of a finite length sequence.[2] The DFT for a sequence, x(nT), is given by:[3]

Equation 1

$$X(kF) = \sum_{n=0}^{N-1} x(nT)e^{-j2\pi kFnt}$$

where
   N = number of samples
   F = spacing of frequency
      domain samples
   T = sample period in the
      time domain

The FFT is simply an efficient algorithm for computing the DFT. Actually, there are several FFT algorithms. Infiniium uses a radix-2 FFT algorithm for computing the DFT. A radix-2 FFT is computed on a number of points equal to a power of 2. The efficiency of the FFT is often expressed in terms of the number of complex multiplications. The number of complex multiplications for a radix-2 FFT can be shown to be $N\log_2 N$. This is a big improvement over the number of computations for a DFT, which is approximately N^2. For example, for a one-million-point sequence, the FFT takes 0.002% of the DFT computation time.[2]

The spacing of the frequency-domain samples or *bins* in the DFT is given by the following equation:

Equation 2

$$F = \frac{1}{NT} = \frac{F_s}{N}$$

where
   F_s = sampling frequency

Thus, the frequency resolution can be improved by increasing N or decreasing F_s.

The DFT is symmetrical about N/2. The magnitude of the DFT is an even symmetric function and the phase is an odd symmetric function. Infiniium plots only the first half of the FFT points since no additional information is provided by the remaining points. The frequency of a particular FFT point, k, is:

Equation 3

$$F_k = \frac{kF_s}{N}$$

The maximum frequency plotted is $F_s/2$, where k = N/2.

FFT Fundamentals

Discrete Fourier Transform continued

The DFT is a complex exponential from which both magnitude and phase can be computed. Infiniium has functions for computing both the magnitude and phase. The phase is computed in degrees. The magnitude is computed in dBm.[3] The voltage form for dBm is:

Equation 4

$$P_{(dBm)} = 20log(V_{RMS} / V_{REF})$$

where
$V_{REF} =$
$\sqrt{0.001\ watts\ *\ 50\ \Omega} = 0.2236\ volts$

The reference voltage, V_{REF}, is defined as the voltage that produces 1 milliwatt of power into 50 Ω. For example, if 1 volt dc is connected to Infiniium, then the magnitude of the FFT result at 0 Hz frequency will be approximately 13 dBm:

$$20log(1.0 / 0.2236) = 13.0\ dBm$$

The Infiniium waveform record length captured is generally not a power of 2. However, the radix-2 FFT requires a number of points equal to a power of 2. Infiniium handles this by padding the end of the waveform with zeros to get to the next power of 2 sequence length.

Although zero padding increases the number of points, it does not change the shape of X(F). It simply extends the number of points in the DFT. For example, if zero padding extends the sequence length by a factor of 2, then every other point of the DFT of the zero-padded sequence has the same value as the DFT of the unpadded sequence. Thus, zero padding has the same effect as interpolation: it fills in points between frequency samples, giving a better visual image of the continuous Fourier transform.

FFT Fundamentals

Sampling Effects

The Fourier transform of an analog signal, $x_a(t)$, is defined by:[2]

Equation 5

$$X_a(F) = \int_{-\infty}^{\infty} x_a(t) e^{-j2\pi Ft} dt$$

To obtain a digital representation, $x(n)$, of an analog signal, $x_a(t)$, a digital oscilloscope samples the signal at uniform intervals, T:

Equation 6

$$x(n) = x_a(t) \big|_{t=nT}$$

Equation 6 assumes the sampling process is ideal such that there is no voltage quantization or other distortion. The Fourier transform of this ideal discrete time sequence is:[2]

Equation 7

$$X(F) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_a\left[\frac{1}{T}(FT + k)\right]$$

Equation 7 shows the relationship between the Fourier transform, $X_a(F)$, of the continuous signal, and the Fourier transform, X(F), of the discrete time sequence. X(F) is the sum of an infinite number of amplitude-scaled, frequency-scaled, and translated versions of $X_a(F)$.
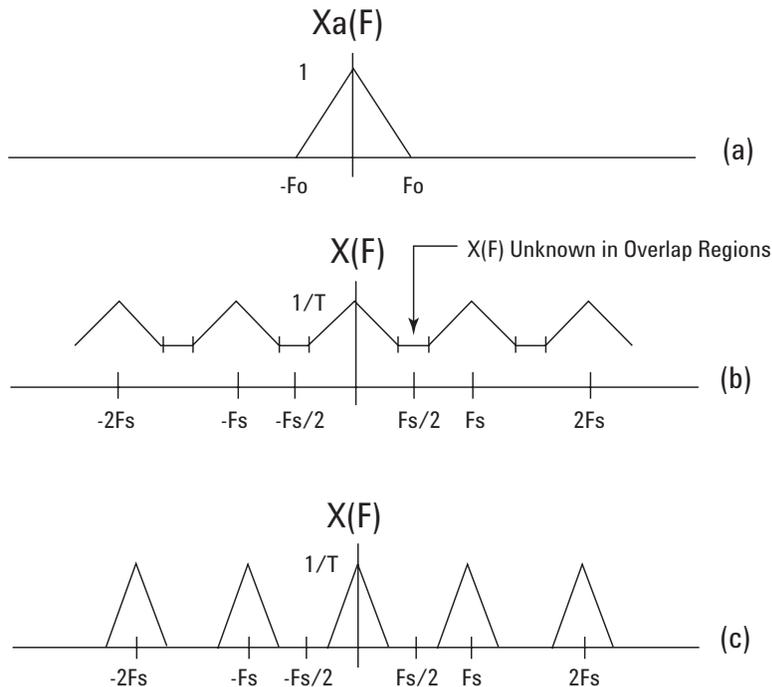
Figure 1. Fourier transform of (a) continuous signal, (b) discrete-time signal with overlap, and (c) discrete-time signal without overlap

Figure 1 (a) shows the Fourier transform, $X_a(F)$, of a continuous signal. Figure 1 (b) and (c) show the Fourier transforms, X(F), of two discrete-time signals obtained by periodic sampling. Notice in (b) and (c) that X(F) is periodic with period 1/T or F_s. Also notice that if F_o is greater than $F_s/2$, as shown by (b), the periodic repetitions of the continuous-time transform overlap, and it is not possible to recover $X_a(F)$ from X(F). When overlap occurs, high-frequency components of $X_a(F)$ fold into lower frequencies in X(F). This effect is called *aliasing*. The frequency at which aliasing occurs, $F_s/2$, is called the *folding frequency*. The maximum frequency in $X_a(F)$ is called the *Nyquist frequency*. The *Nyquist rate* is the minimum sampling rate required to prevent aliasing and is twice the Nyquist frequency.[2]

FFT Fundamentals

Sampling Effects continued

It is insightful to consider aliasing from a time-domain perspective. Assume a sine wave signal is sampled uniformly. If there are less than two samples per period, then it is not possible to determine the frequency of the continuous sine wave signal from the sampled version. In this case, the sampled version will appear to be a sine wave at a lower frequency.

If the sampling rate is at least twice the highest frequency content in the analog signal, no aliasing will occur, and the DFT of the sampled sequence will provide a good estimate of the Fourier transform of the analog signal.

However, if there is frequency content above the folding frequency then these higher frequency components will alias. This effect is shown in figure 2. Two sine waves with frequencies of 3.3 MHz and 6.1 MHz are digitized by Infiniium with a sampling rate of 5 MSa/s. The FFT magnitude of the resulting digital waveform is shown. Because both sine wave frequencies are above the folding frequency of 2.5 MHz, they are aliased to lower frequencies below the folding frequency.

Equation 8 can be used to compute the alias frequency, F', from the original frequency, F:

Equation 8

$$F' = \frac{F_s}{2} - \left| F \ modulus \ F_s - \frac{F_s}{2} \right|$$

Using this equation, the alias frequency for F = 3.3 MHz is 1.7 MHz and the alias frequency for F = 6.1 MHz is 1.1 MHz. Notice that the spectral lines shown in figure 2 agree with these calculations.
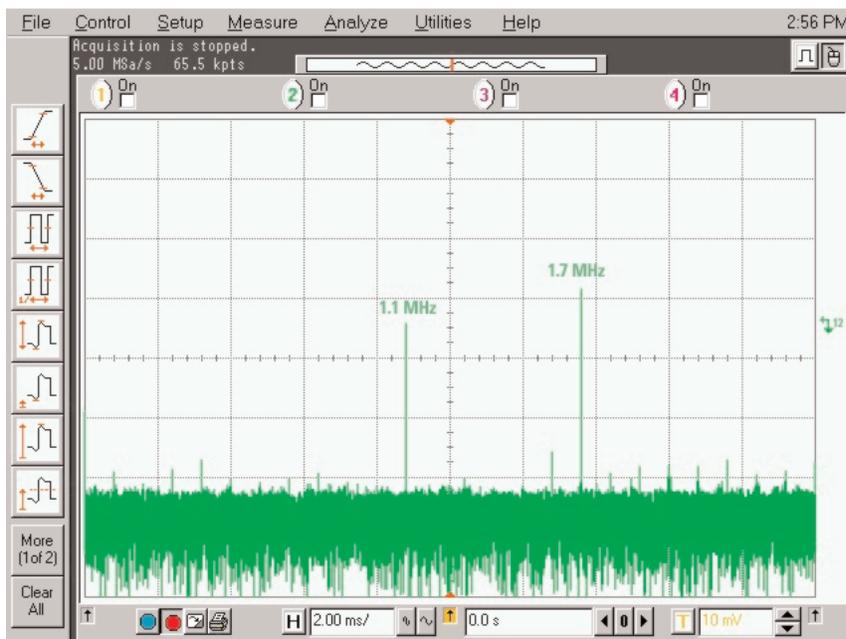
Acquisition	Sampling mode real time Normal Configuration 4GSa/s
	Memory depth manual Memory depth 65536pts
	Sampling rate manual Sampling rate 5.00 MSa/s
	Averaging off Interpolation on
Channel 1	Scale 1.00 V/div Offset 0.0 V
	BW limit off Coupling DC Impedance 1 M Ohm
	Attenuation 1.000 : 1 Atten units ratio Skew 0.0 s
	Ext adapter None Ext coupler None
	Ext gain 1.00 Ext offset 0.0
Time base	Scale 2.00 ms/ Postion 0.0 s Reference center
Trigger	Mode edge Sweep auto
	Hysteresis normal Holdoff time 80 ns Coupling DC
	Source channel 1 Trigger level 10 mV Slope rising
Function 2	FFT magnitude channel 1
	Vertical scale 20.0 dBm/div Offset -13.0000 dBm
	Horizontal scale 250 kHz/div Position 1.25000 MHz
	Window Hanning Resolution 76.2939 Hz

Figure 2. Infiniium FFT magnitude plot of 3.3 MHz and 6.1 MHz sine wave inputs

FFT Fundamentals

Spectral Leakage and Windowing

The DFT is computed on a finite length sequence. It attempts to approximate the Continuous Fourier Transform (CFT), which integrates over all time. The DFT is a sampled version of the CFT of an infinite sequence formed by replicating the finite sequence an infinite number of times.

If the end points of the finite sequence do not match, then the infinite sequence will have a discontinuity between adjacent periods of the finite sequence. This discontinuity in the time domain causes *spectral leakage* in the frequency domain. In general, when a digital oscilloscope captures a signal it does not capture an integral number of periods, which introduces a discontinuity in the periodically extended signal. Thus, the FFT computed by the digital oscilloscope almost always shows spectral leakage. The spectral leakage causes the fine spectral lines to spread out into wider lobes.

Another way to explain spectral leakage is to consider the finite sequence as a product of two infinite length sequences: the original sequence and a window sequence. The window sequence is nonzero for a finite number of samples and zero elsewhere. By multiplying the infinite sequence by the window sequence, the window extracts a finite chunk of the infinite sequence. Multiplication in the time domain corresponds to convolution in the frequency domain. So the resulting Fourier transform of the finite length sequence is just the Fourier transform of the infinite sequence convolved with the Fourier transform of the window sequence.
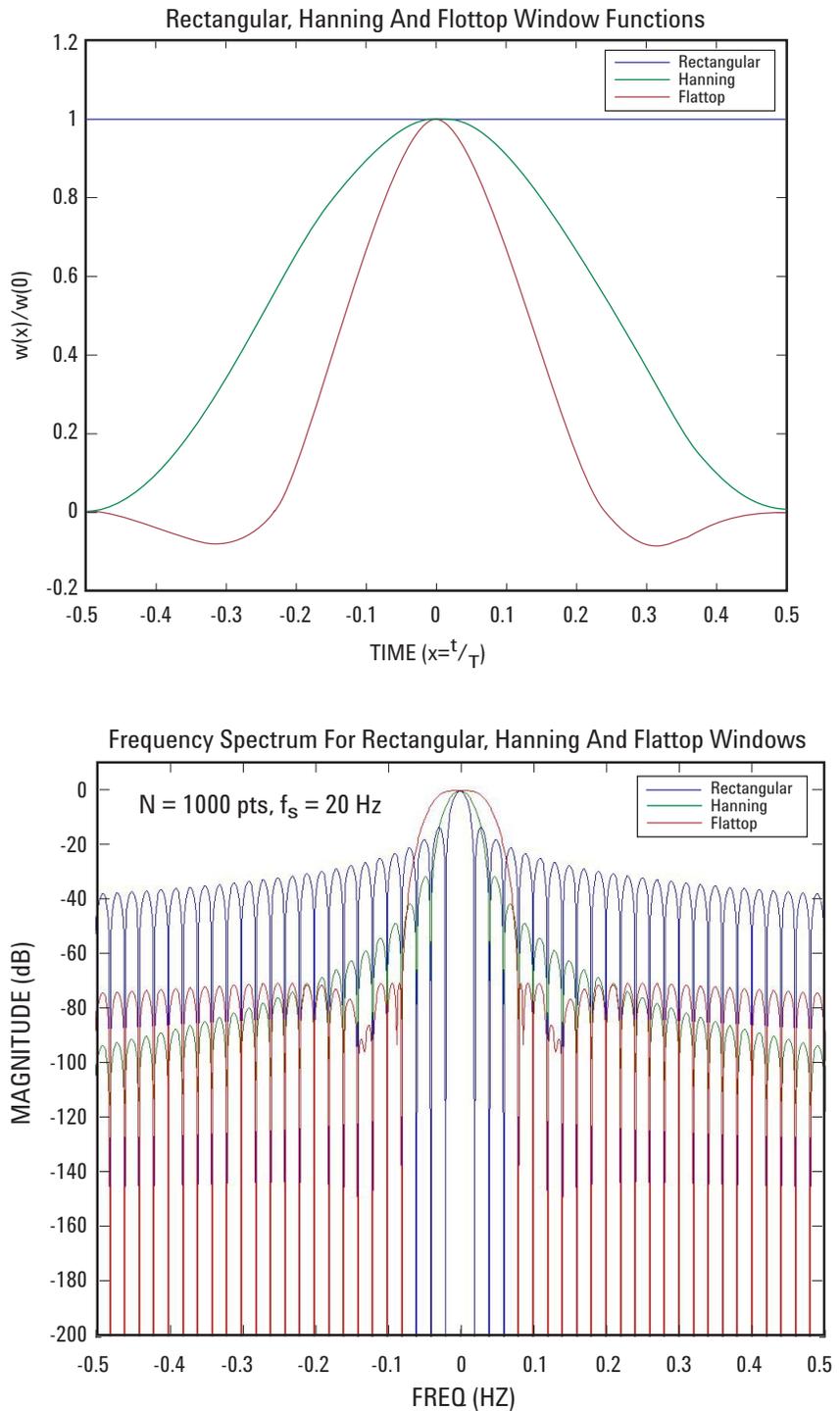
Figure 3. Infiniium FFT window functions (top) and corresponding frequency spectrums (bottom)

FFT Fundamentals

Spectral Leakage and Windowing continued

This convolution causes each spectral line in X(F) to take on the shape of the Fourier transform of the window sequence.

The shape of the window determines the shape of the spectral lobes in the Fourier transform. The Fourier transform for a rectangular window is a *sinc* function.[2] The rectangular window does nothing to reduce the discontinuity in the periodic extension of the finite time sequence. However, other windows have been defined that approach the zero value smoothly on each end.

Infiniium provides three choices for window functions, *rectangular*, *Hanning*, and *flattop*. These windows and the corresponding Fourier transforms are shown in figure 3. Notice that the rectangular window has the narrowest main lobe, but the side lobes fall off gradually. The width between both zeros of the main lobe is 2 bins, and the first side lobe is 13.3 dB below the main lobe. These side lobes obscure neighboring spectra. The Hanning window has a wider main lobe, but the side lobes fall off more sharply. The main lobe width is 4 bins, and the first side lobe is 31.5 dB below the main lobe. The flattop window has the widest main lobe, 8 bins, and the sharpest side lobe fall off. The first side lobe is 70.4 dB below the main lobe.

The DFT represents discrete samples from the continuous Fourier transform, X(F), of the discrete-time sequence. The interval between frequency bins in the DFT is given by equation 2. When an integral number of periods of the continuous-time signal are captured in the discrete-time sequence, the center of a frequency bin in the DFT lines up with the center of the main lobe, and the side lobe leakage disappears. When a sequence is extended by zero padding, the DFT bin spacing gets smaller and more of the true window shape is revealed. As stated earlier, zero padding does not change the shape of X(F), it simply provides more samples in the DFT, revealing more of X(F).

Practical Considerations

Frequency Span and Resolution

The previous sections described several properties and characteristics of the FFT and sampling that effect the accuracy of the frequency spectral estimate. This section focuses on the practical aspects of using an Infiniium deep-memory oscilloscope for spectral analysis. The benefits of deep memory on FFT resolution and dynamic range are described in detail. Several characteristics of the oscilloscope effect the dynamic range. These factors and using averaging to improve the dynamic range are described, as well as methods for selecting the proper window function for particular types of analysis. Also, this section explains how to configure the oscilloscope time range, sampling rate and memory depth to get the desired FFT frequency span, resolution and update rate.

The maximum sampling rate currently available from the Infiniium deep-memory oscilloscopes is 4 GSa/s, which allows for FFT-based spectral analysis up to 2 GHz. The maximum memory depth currently available is 16 MSa. When the full sampling rate and memory depth are employed, Infiniium can provide a frequency span of 2 GHz with a frequency resolution of 250 Hz.

It is important to realize that Infiniium computes the FFT for those time-domain samples that fall onscreen only. Offscreen samples are ignored. To get all the time-domain samples onscreen for a manual sampling rate setting, the user simply adjusts the horizontal scale for a longer time range.

Both sampling rate and memory depth can be controlled manually. In automatic mode, the memory depth is adjusted to get the maximum sampling rate possible for the current time range. Although this is a convenient feature for time-domain analysis, it is not always what the user wants for frequency-domain analysis. In many cases, the user will purposely reduce the sampling rate to improve the frequency resolution. The user may also reduce the memory depth to improve the FFT update rate.

The frequency resolution of the FFT depends on both the sampling rate and the number of points, as shown by equation 2. The resolution can be improved by either increasing the number of points or decreasing the sampling rate. However, decreasing the sampling rate reduces the maximum frequency computed and opens the door to additional aliasing. The FFT frequency resolution can be easily determined from the time range setting of the oscilloscope, as shown by the following equation:

Equation 9

$$F = \frac{1}{NT} = \frac{1}{Time\ Range}$$

From the information presented earlier, it is now possible to determine the frequency resolution and span of the Infiniium FFT for a particular sampling rate and record length.

For example, assume the sampling rate is set to 4 GSa/s and the memory depth is set to 100,000 points. In this case, Infiniium will zero-pad the number of points to the next power of 2 or 131,072 points. The frequency span displayed will cover half the sampling frequency or 2 GHz. The frequency resolution will be 4 GSa/s divided by 131,072 or 30.5176 kHz. Since the FFT is computed only for those points that fall onscreen, to get all the points onscreen, the horizontal scale must be set to 2.5 µs/div or slower. The horizontal scale setting required to get all the data onscreen for a particular sampling rate and memory depth is shown by the following equation:

Equation 10

$$Horizontal\ Scale \geq \frac{N}{10F_s}\ s\,/\,div$$

Generally, it is not necessary to apply this equation in practice. The user simply adjusts the time range until all the data is onscreen. The memory bar displayed at the top of the screen simplifies this adjustment.

Practical Considerations

Frequency Span and Resolution continued

Figure 4 shows the impact of deep memory on the resolution of the FFT spectrum. The spectrum shown is produced from a signal containing two sine waves of nearly identical frequencies, F1 and F2. F1 is 75.000273 MHz and F2 is 75.001546 MHz. The frequency delta between F1 and F2 is 1.273 kHz.

The sampling rate is set at 200 MSa/s, which is more than the Nyquist rate of 150.003092 MHz. The sampling rate is selected as low as possible to improve the FFT resolution, but high enough to prevent aliasing. The Hanning window function is used to reduce side lobe interference.

With 256 kPts, the FFT resolution is 762.9 Hz, which, because of spectral leakage, is not enough to differentiate the two signals. However, with a memory depth of 4 MPts, the FFT resolution is 47.7 Hz, and the spectra from both signals are clearly visible.

Deep memory doesn't come for free. Increasing the memory depth slows down the FFT update rate. As stated previously, the number of complex multiplications required to compute an N-point FFT is $Nlog_2(N)$. This dominates the computation time.

Currently, the Infiniium deep-memory oscilloscope can compute and display a one-million-point FFT in one second. A 16-million-point FFT, however, takes 20 seconds. In automatic memory-depth mode, Infiniium limits the number of points for an FFT function to one million so that the update rate is no slower than one update per second. To optimize the memory depth for a particular situation, the manual memory control should be selected.
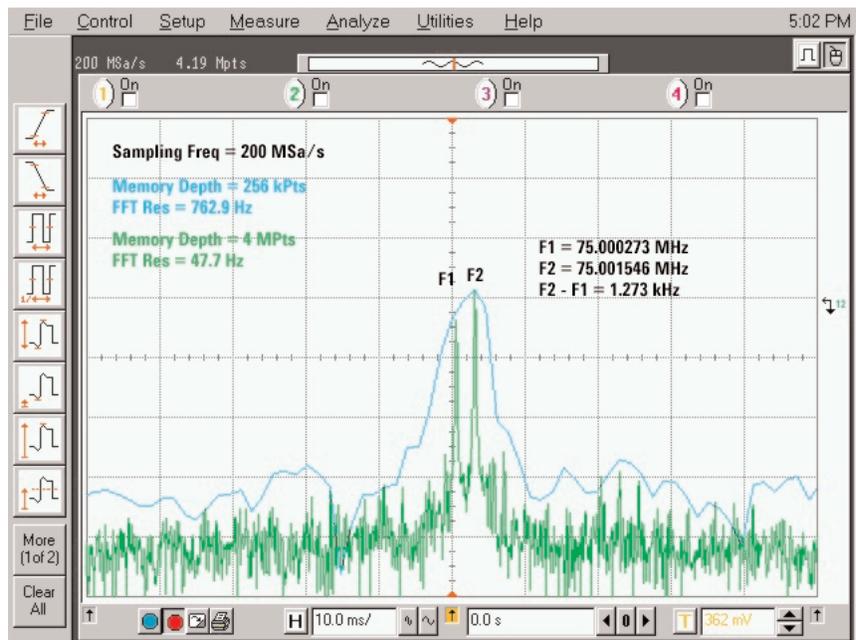
Figure 4. Infiniium FFT spectrum showing the impact of deep memory on improved FFT resolution

Practical Considerations

Dynamic Range

The dynamic range of a frequency-domain measurement is an indication of the smallest signal that can be viewed in the presence of a larger signal. It indicates the magnitude delta between the fundamental spectra of a full-scale input and the noise floor.

The dynamic range of an FFT spectrum, produced by a digital oscilloscope, is limited by analog-to-digital converter (ADC) quantization and other internal oscilloscope noise sources. Also, nonlinear responses from the front-end amplifiers and the ADC produce harmonic distortions that show up as artifacts in the FFT spectrum.[3] These factors limit the user's ability to detect spectral information in the signal and differentiate it from the oscilloscope. The next section will show how averaging can improve the dynamic range.

Figure 5 shows the frequency spectrum produced by the Infiniium FFT for a pure 470 MHz sine wave input. Notice all the spectral lines showing up at frequencies other than the fundamental 470 MHz. Harmonics of 470 MHz at 940 MHz, 1410 MHz, and 1880 MHz can be seen, as well as harmonics above the folding frequency of 2 GHz, which alias to lower frequencies in the spectrum. The 1650-MHz spectral line is one such aliased harmonic of the 470-MHz input. Spectral artifacts from an internal 125-MHz clock also show up in the spectrum. The dynamic range between the 470-MHz fundamental and the largest spectral artifact is 51 dB.

Increasing the memory depth reduces the noise floor of asynchronous noise sources and improves the dynamic range. This improvement occurs because increasing the number of FFT points does not change the total noise power.

For example, doubling the number of FFT points drops the magnitude of the asynchronous noise components by a factor of 2 so that the total noise sum remains constant. On a log scale, doubling the number of FFT points reduces the noise floor by 3 dB. This expected improvement drops off somewhat for large record lengths because of round-off errors in computing the FFT.
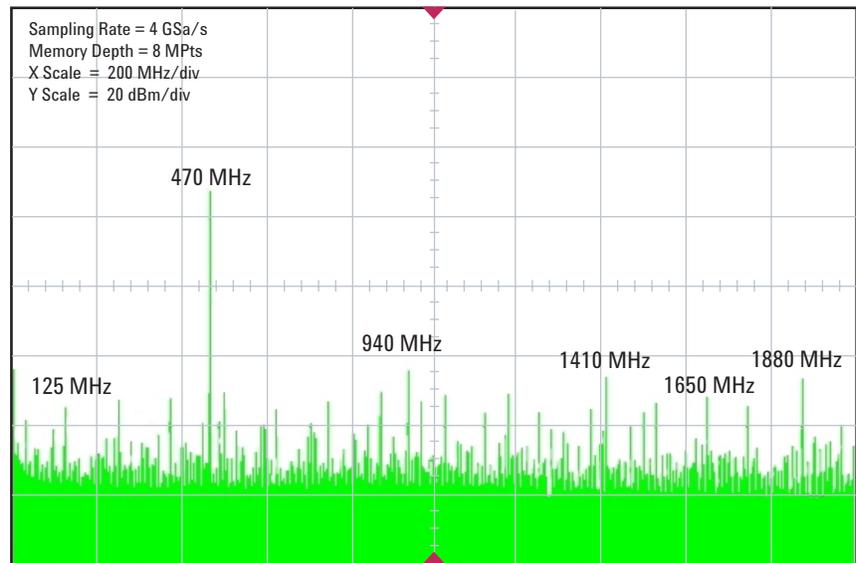
Figure 5. Infiniium FFT spectrum with 470 MHz pure sine wave input

Practical Considerations

Dynamic Range continued

Figure 6 shows the effect of increasing the FFT record length on dynamic range. The blue FFT trace is acquired on an Infiniium with a memory depth of 1024 points. The green FFT trace is acquired with a memory depth of 16,400,000 points. Both are acquired at 4 GSa/s. Notice that the noise floor drops about 30 dB for the deep-memory FFT, revealing spectral data that are masked with only 1024 points. Unfortunately, some of the additional spectral lines that become visible with deep memory are due to the oscilloscope and are not actually present in the signal.

Digital oscilloscopes sometimes include *effective bits* in their specifications. Effective bits is a measure of the *signal-to-noise ratio* (SNR). SNR is the ratio of the signal power to the total noise power. It is typically computed in the time domain using sine wave curve fitting.[4] SNR can be calculated from effective bits using the following equation:[5]

Equation 11

$$SNR = Eff\ Bits\ *\ 6.02 + 1.8\ dB$$

The effective bits measurement is typically worse with fast-slewing inputs, so it is often specified at both high and low frequencies. Infiniium uses an 8-bit ADC. Using equation 11, an ideal 8-bit ADC has an SNR of 50 dB.

Sampling Rate = 4 GSa/s
Vertical Scale = 20 dBm/div

Memory Depth = 1024 pts
Memory Depth = 16,400,000 pts

250 MHz

Figure 6. Infiniium FFT spectrum showing how deep memory reduces the noise level

Practical Considerations

Dynamic Range continued

Figure 7 shows a 1024-point FFT spectrum for an ideal 8-bit ADC with a full-scale input. Notice with 1024 points an SNR of 50 dB provides a dynamic range of about 70 dB. In contrast, as shown by the 1024-point FFT in figure 6, Infiniium has a dynamic range of about 50 dB for a 250 MHz, full-scale sine wave input. This reduction of 20 dB from the ideal is caused by oscilloscope noise sources and nonlinear effects.

1024 Pt. FFT Spectrum for Ideal 8-bit ADC
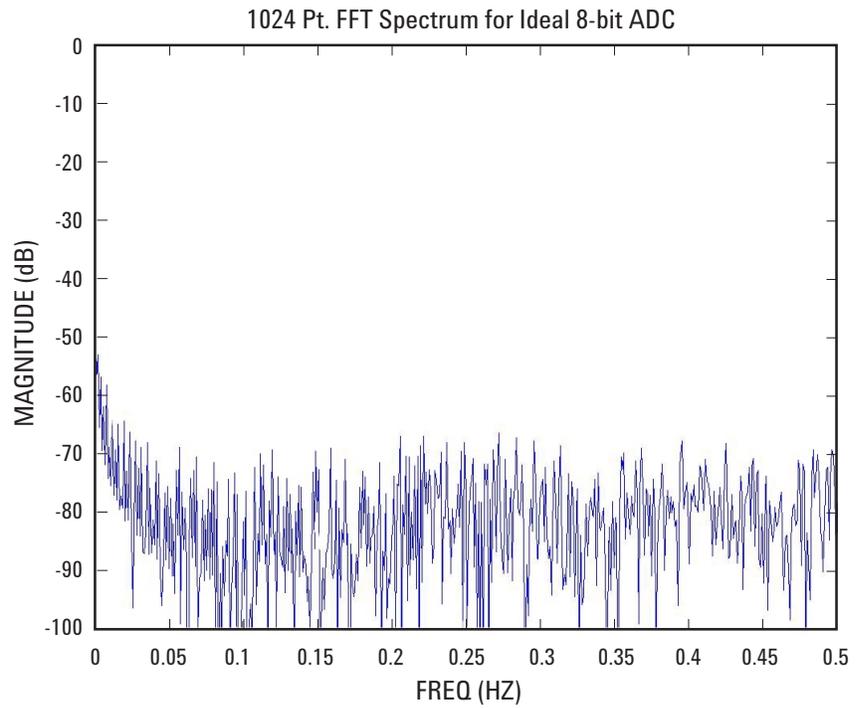
Figure 7. Compare this 1024-point FFT spectrum for an ideal 8-bit ADC with the 1024-point FFT in figure 6

Practical Considerations

Averaging

This section describes two types of averaging that Infiniium can perform. One type of averaging is applied to time-domain wave-forms prior to computing the FFT. It is useful for reducing noise to improve the dynamic range. The other type of averaging is applied to the FFT magnitude to reduce the variance of the spectrum estimate.

Time-domain averaging attenuates asynchronous noise sources by averaging time-domain wave-forms from multiple triggers. Signal bandwidth is not effected. The signal must be periodic to take advantage of this type of averaging. Noise variance is reduced by a factor equal to the number of averages. In terms of decibels, time-domain averaging reduces the noise floor by:

Equation 12

$$Noise\ Floor\ Drop =$$
$$3\ dB * log_2(N_{avg})$$

where
N_{avg} is the number of averages

For Infiniium, the number of averages is selectable from 2 to 4096. The maximum memory depth available, with averaging enabled, is 64 kPts. If larger memory depth is required, the average function can be applied to the time-domain waveform, but it runs much slower.

Time-domain averaging is useful, not only for reducing signal noise, but also for reducing internal, asynchronous noise sources from the oscilloscope. Figure 8 shows the impact of time-domain averaging on dynamic range. The blue trace is the FFT spectrum with no averaging and the green trace is the FFT spectrum with 256 averages. The black regions indicate where the two traces overlap. For both spectra, the sampling rate is 4 GSa/s and the memory depth is 64 kPts. The vertical scale is 20 dB/div.

Notice that 256 averages reduce the noise floor by about 24 dB, as expected from equation 12. No signal is input to the oscilloscope, so what is shown results from internal noise sources in the oscilloscope.

The average value for the DFT spectrum converges to the true value as the number of points increases. However, the variance remains constant.[6] To improve the variance, multiple DFT spectrums can be averaged together. The variance is reduced by a factor equal to the number of averages.

By chaining the average function to the FFT magnitude function, Infiniium provides a way to average FFT spectrums. The averaging is performed on the decibel magnitude values. The number of averages can be set from 2 to 4096.
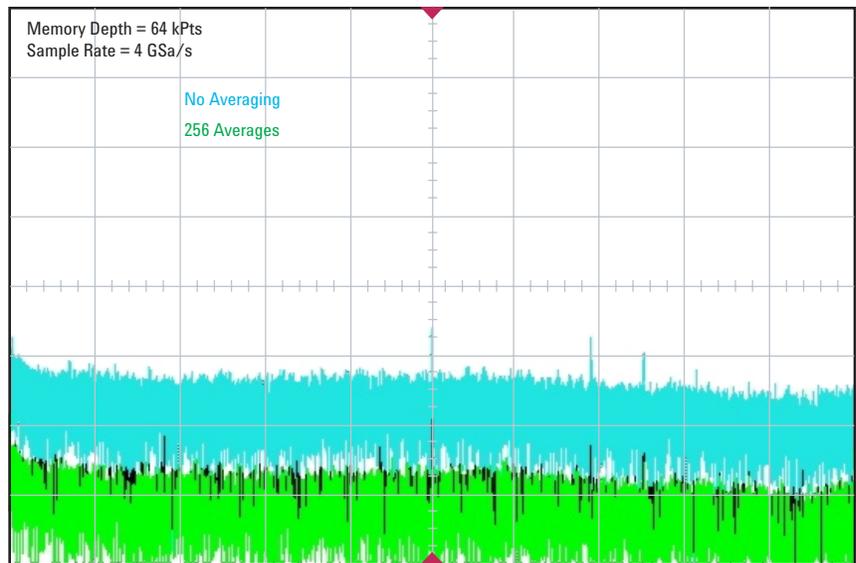
Figure 8. Infiniium FFT spectrum showing the impact of averaging time-domain waveforms for increasing dynamic range

Practical Considerations

Averaging continued

Figure 9 shows the effect of averaging FFT spectrums. The green spectrum trace, function 2, is not averaged. The purple spectrum trace, function 3, results from averaging function 2 sixteen times. The black regions indicate where function 2 and function 3 overlap. It is clear from figure 9 that this type of averaging reduces the variance of the FFT spectrum but does not improve dynamic range.

Memory Depth = 32 kPts

Non-averaged FFT Magnitude

Averaged FFT Magnitude, 16 Averages
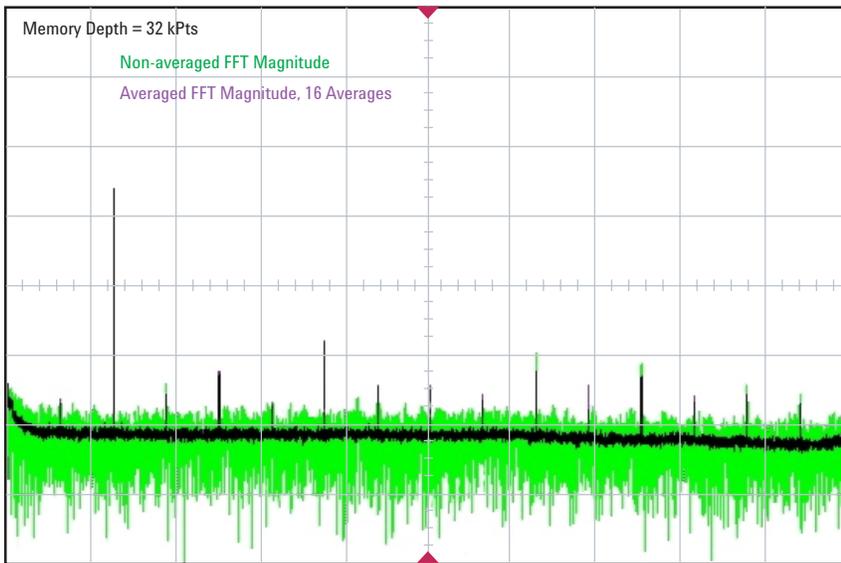
Figure 9. Averaged FFT spectrums reduce variance

Practical Considerations

Window Selection

Infiniium provides three window functions to choose from, rectangular, Hanning, and flattop (refer to figure 3). By selecting the proper window function, more useful information can be obtained from the FFT spectrum.

Although the rectangular window has the narrowest main lobe, it is not generally used because the side lobes fall off gradually, obscuring neighboring spectra. The Hanning window is the most common window used for viewing spectral content. The main lobe width is twice that of the rectangular window, but the side lobes fall off sharply.

When making magnitude measurements on spectral data, the flattop window is the best choice. Because of the broad flat top it exhibits in the frequency domain, the amplitude of a spectral peak is very accurate. The maximum amplitude error due to spectral leakage, using the flattop window, is 0.1 dB. This is much better than the maximum error for the Hanning window, which is 1.5 dB.[3] The maximum error occurs when the center of the main lobe falls exactly halfway between two frequency bins. However, the flattop window is not always a good choice when trying to resolve closely-spaced spectral lines because of the broad main lobe.

Increasing the memory depth improves the frequency resolution of the FFT without reducing the frequency span (refer to equation 2). In terms of frequency bins, the window function lobe width is constant. But the frequency width of each bin is inversely proportional to the number of points. So increasing the number of points decreases the spectral line lobe widths and improves the ability to resolve closely-spaced spectral lines. For example, by doubling the memory depth, the flattop window resolves as well as the Hanning window does with half the memory depth.

Practical Considerations

Equivalent Time Sampling

Aliasing is always a potential issue for a digital oscilloscope. The oscilloscope passes all frequencies out to the specified analog bandwidth. The rolloff is not necessarily sharp, so that even frequencies beyond the bandwidth can be passed on to the ADC.

Without a low-pass cutoff filter, frequencies above the folding frequency are allowed to alias into the frequency spectrum. The problem is more severe when the sampling rate is reduced. Ideally, for spectral analysis, a low-pass analog filter would be injected to attenuate all frequencies above the folding frequency. This, however, is not practical for a digital oscilloscope.

Equivalent time sampling can be used to increase the effective sampling rate and reduce aliasing. Infiniium uses a form of equivalent time sampling referred to as *random repetitive sampling*. The signal must be periodic for this sampling technique to work properly. A random repetitive record is built up over several acquisitions. The time between the sample clock and the trigger event is carefully measured and used to place the samples from each acquisition. The accuracy of measuring this time dictates how closely the samples can be spaced. The inverse of the sample spacing is the *effective sampling rate*.

For the Infiniium deep-memory oscilloscope, the minimum sample spacing in equivalent time is 4 ps, providing a maximum effective sampling rate of 250 GSa/s.

There are several aspects to equivalent time sampling in Infiniium that limit its usefulness for spectral analysis. First, the user has no direct control over the effective sampling rate. Also, the maximum memory depth is limited to 32 kPts. Finally, the sample interval is not uniform, which introduces additional noise to the frequency spectrum. Samples can fall anywhere within one *time bucket*, but for purposes of computing the FFT they are assumed to be evenly spaced.

Figure 10 shows the FFT spectrum for a pure 470-MHz sine wave input using equivalent time sampling. Because of the high effective sampling rate and limited memory depth, the frequency resolution of 7.63 MHz is too large to resolve closely-spaced spectral

lines. Also, the dynamic range between the fundamental and the highest spectral harmonic is only 38 dB.

Comparing this FFT spectrum to that achieved with real-time sampling, figure 5, it is clear that real-time sampling provides better dynamic range and frequency resolution.

To improve the frequency resolution for equivalent time, the effective sample period can be increased by increasing the time range. However, because the sampling is not uniform, additional jitter is introduced between samples, further eroding the useful dynamic range. Time-domain averaging can be used to reduce this jitter and improve the dynamic range.

470 MHz

Equivalent Time FFT
Effective Sampling Rate = 250 GSa/s
Memory Depth = 32768 Pts
Frequency Resolution = 7.63 MHz
Freq Span = 2 GHz
Y Scale = 20 dBm/div
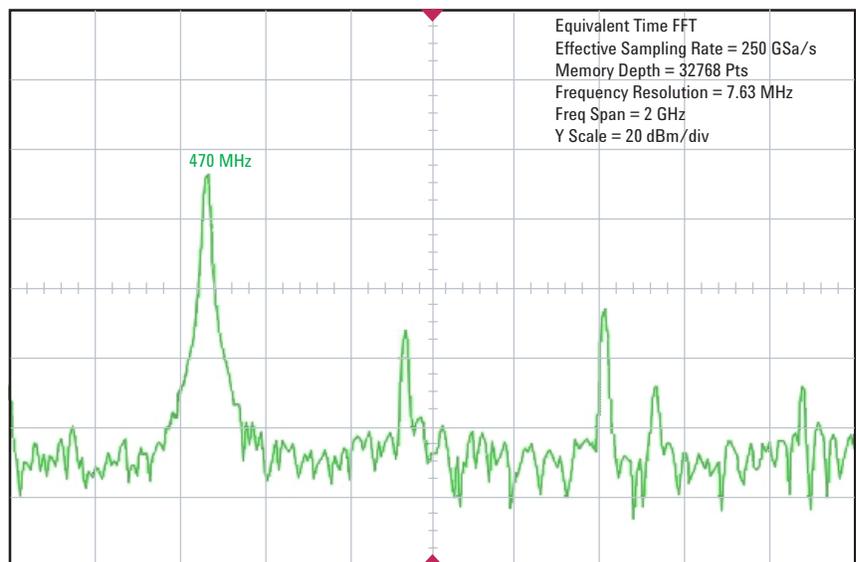
Figure 10. Infiniium FFT spectrum for 470 MHz sine wave input using equivalent time sampling to increase the effective sampling rate and reduce aliasing

Application

Characterizing an AM Signal

This section describes a real-world application for a deep-memory oscilloscope FFT. The application involves measuring the characteristics of an amplitude-modulated (AM) signal. The characteristics of interest are the carrier frequency, f_o, modulation frequency, f_m, and modulation index, a.[3]

The spectrum of an AM signal contains all the information necessary to compute these parameters. Figure 11 shows the spectrum of a typical AM signal with sinusoidal modulation. The center spectral line represents the carrier and the sidebands result from the modulation. The modulation frequency is the difference between the carrier frequency and one of the sidebands. The modulation index is a measure of the amplitude difference between the carrier signal and the modulation signal. It can be computed from the magnitude delta, A_{dB}, between the carrier and modulation sidebands, and is given by the following equation:

Equation 13

$$a = 2x10^{(A_{dB} / 20)}$$

When the modulation frequency is a small percentage of the carrier frequency, a high resolution FFT spectrum is necessary to distinguish the sidebands from the carrier.
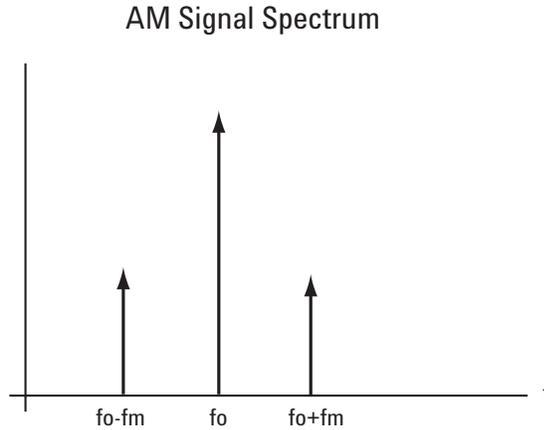
AM Signal Spectrum

Figure 11. The spectrum of an AM signal with sinusoidal modulation

For this example, an Agilent Technologies 33250A function generator is used to generate an AM signal with the following parameters:

- Carrier frequency = 77 MHz
- Modulation frequency = 1 kHz
- AM depth = 2%
- Shape = Sine

where the AM depth is equal to the modulation index. Notice that the modulation frequency is 0.0013% of the carrier frequency. Differentiating the modulation from the carrier will require a high-resolution frequency spectrum.

Application

Selecting Sampling Rate and Memory Depth

The first task in setting up the oscilloscope to make this measurement is to select the sampling rate and memory depth. To prevent aliasing, the sampling rate, F_s, is set to a value greater than twice the Nyquist frequency of 77 MHz + 1 kHz. Also, for better resolution, the minimum available sampling rate that meets this criteria is selected. For Infiniium, the next available sampling rate, which is greater than twice the Nyquist frequency, is 200 MSa/s.

To obtain the most accurate measurement of the modulation index, a flattop window is used. The flattop window is 8 bins wide, so to clearly distinguish the modulation sidebands requires a frequency resolution of 1 kHz divided by 8. Using this fact and referring to equation 2 for computing the frequency resolution, the following equation for computing the minimum number of points is derived:

Equation 14

$$N \geq \frac{F_s}{F} =$$

$$\frac{F_s}{\frac{f_m}{W}} = \frac{200\ MSa/s}{\frac{1\ kHz}{8}} = 1.6\ MPts$$

where
   F is the frequency resolution
   and W is the window main-
   lobe-width in bins

For this example, since the FFT update rate is not an issue, the memory depth is set to 8 MPts, which exceeds the minimum requirement and provides a frequency resolution of 23.8 Hz.

Scaling Time and Frequency

The next task is to scale the oscilloscope time and frequency axes. Recalling that the FFT is only computed for onscreen points, the memory bar at the top of the screen is used to set the time range so that all the captured data is onscreen. Using equation 10, the time/div must be set to a value greater than or equal to 4 ms/div.

$$Horizontal\ Scale \geq$$

$$\frac{8\ MPts}{10\ x\ 200\ MSa/s} = 4ms\ /\ div$$

In this example, the time/div is set to 5 ms/div. The frequency axis is scaled so that the carrier frequency is centered and the sidebands are separated from the carrier by one division.

Application

Resulting FFT Spectrum

Figure 12 shows the FFT spectrum for the AM signal captured by Infiniium. Notice that the sidebands are clearly distinguishable from the carrier. Also notice that the dynamic range is adequate to distinguish the sidebands from the noise floor, even with a modulation index as small as 2%. The listing below the screen image shows all the setup information and measurement results. A delta peak frequency measurement is used to find the modulation frequency. The carrier frequency can be read directly from the center peak marker. A delta peak magnitude measurement is used to find the magnitude delta between the carrier peak and the right sideband. The measurement result is -40.17 dB. Plugging this value into equation 13 gives the modulation index:

$$a = 2 x 10^{(-40.17\ dB\ /\ 20)} = 1.96\%$$

The AM signal parameters measured from the FFT spectrum are summarized as follows:

• Carrier frequency = 77.0002 MHz
• Modulation frequency = 1.001 kHz
• Modulation Index = 1.96%

Comparing these results with the signal generator setup given earlier shows that the measurement error is small and the results are excellent. In this example, a single acquisition is used to make the measurements. Because Infiniium keeps track of the mean and standard deviation for measurements, the measurement results can be improved by running several acquisitions.
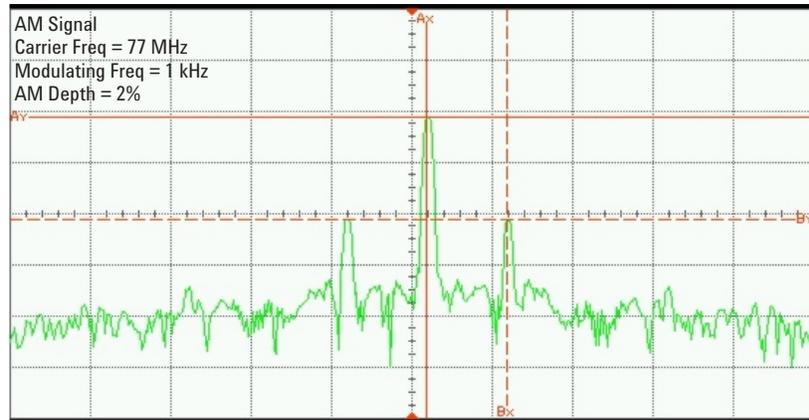
AM Signal
Carrier Freq = 77 MHz
Modulating Freq = 1 kHz
AM Depth = 2%

Acquisition	Sampling mode real time Normal Configuration 4GSa/s
	Memory depth manual Memory depth 8200000pts
	Sampling rate manual Sampling rate 200 MSa/s
	Averaging off Interpolation on

Channel 1	Scale 20 mV/div Offset 2 mV
	BW limit off Coupling DC Impedance 50 Ohms
	Attenuation 1.000 : 1 Atten units ratio Skew 0.0 s
	Ext adapter None Ext coupler None
	Ext gain 1.00 Ext offset 0.0

Time base	Scale 5.00 ms/ Postion 0.0 s Reference center

Trigger	Mode edge Sweep auto
	Hysteresis normal Holdoff time 80 ns Coupling DC
	Source channel 1 Trigger level 1.6 mV Slope rising

Function 2	FFT magnitude channel 1
	Vertical scale 20.0 dBm/div Offset -51.0000 dBm
	Horizontal scale 1.00 kHz/div Position 77.0000 MHz
	Window flattop Resolution 23.8419 Hz

Measure		current	mean	std dev	min	max
	FFT Δfreq(f2)	1.001 kHz	1.001 kHz	0 Hz	1.001 kHz	1.001 kHz
	FFT Δmag(f2)	-40.17 dB	-40.17 dB	0.0 dB	-40.17 dB	-40.17 dB

Marker		current	mean		X	Y
	FFT Δfreq(f2)	1.001 kHz	1.001 kHz	A—(f2) =	77.000189 MHz	-13.27 dBm
	FFT Δmag(f2)	-40.17 dB	-40.17 dB	B---(f2) =	77.001191 MHz	-53.44 dBm
				Δ =	1.001 kHz	-40.17 dB
				1/ΔX =	999 µs	

Figure 12. Infiniium FFT spectrum of an AM signal with sinusoidal modulation

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Your Advantage means that Agilent offers a wide range of additional expert test and measurement services, which you can purchase according to your unique technical and business needs. Solve problems efficiently and gain a competitive edge by contracting with us for calibration, extra-cost upgrades, out-of-warranty repairs, and on-site education and training, as well as design, system integration, project management, and other professional engineering services. Experienced Agilent engineers and technicians worldwide can help you maximize your productivity, optimize the return on investment of your Agilent instruments and systems, and obtain dependable measurement accuracy for the life of those products.

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