Understanding
Reading Accuracy and
Resolution in a 6½-Digit DMM

Application Note

Agilent Technologies

Introduction

This application note explains the reality and the mathematics behind digital multimeter (DMM) resolution and DC noise specifications, which directly correlate to the number of digits available from the DMM at a given resolution and speed. In this paper, we use the 6$\frac{1}{2}$-digit Agilent 34410A DMM as an example.

DMM accuracy and resolution

Figure 1 shows the DC voltage accuracy specifications for the 34410A DMM. On the 10 V range, and with the integration time set to 100 PLC ("power line cycles" — 1 PLC is 16.667 ms

DC Characteristics

Accuracy Specifications (% of reading + % of range)[1]

Function	Range[3]	Test Current or Burden Voltage	24 Hour[2] T$_{CAL}$ ±1ºC	90 Day T$_{CAL}$ ±5ºC	1 Year T$_{CAL}$ ±5ºC	Temperature Coefficient 0ºC to (TCAL –5ºC) (T$_{CAL}$ +5ºC) to 55ºC
DC	100.0000 mV		0.0030+0.0030	0.0040+0.0035	0.0050+0.0035	0.0005+0.0005
	1.000000 V		0.0020+0.0006	0.0030+0.0007	0.0035+0.0007	0.0005+0.0001
	10.00000 V		0.0015+0.0004	0.0020+0.0005	0.0030+0.0005	0.0005+0.0001
	100.0000 V		0.0020+0.0006	0.0035+0.0006	0.0040+0.0006	0.0005+0.0001
	1000.000 V[5]		0.0020+0.0006	0.0035+0.0006	0.0040+0.0006	0.0005+0.0001
Resistance [4]	100.0000 Ω	1 mA current source	0.0030+0.0030	0.008+0.004	0.010+0.004	0.0006+0.0005
	1.000000 kΩ	1 mA	0.0020+0.0005	0.007+0.001	0.010+0.001	0.0006+0.0001
	10.00000 kΩ	100 µA	0.0020+0.0005	0.007+0.001	0.010+0.001	0.0006+0.0001
	100.0000 kΩ	10 µA	0.0020+0.0005	0.007+0.001	0.010+0.001	0.0006+0.0001
	1.000000 MΩ	5.0 µA	0.0020+0.0010	0.010+0.001	0.012+0.001	0.0010+0.0002
	10.00000 MΩ	500 nA	0.0100+0.0010	0.030+0.001	0.040+0.001	0.0030+0.0004
	100.0000 MΩ	500 nA II 10 MΩ	0.200+0.001	0.600+0.001	0.800+0.001	0.1000+0.0001
	1000.000 MΩ	500 nA II 10 MΩ	2.000+0.001	6.000+0.001	8.000+0.001	1.0000+0.0001
DC Current	100.0000 uA	<0.03 V burden V	0.010+0.020	0.040+0.025	0.050+0.025	0.0020+0.0030
	1.000000 mA	<0.3 V	0.007+0.006	0.030+0.006	0.050+0.006	0.0020+0.0005
	10.00000 mA	<0.03 V	0.007+0.020	0.030+0.020	0.050+0.020	0.0020+0.0020
	100.0000 mA	<0.3 V	0.010+0.004	0.050+0.005	0.050+0.005	0.0020+0.0005
	1.000000 A	<0.80 V	0.050+0.006	0.080+0.010	0.100+0.010	0.0050+0.0010
	3.00000 A	<2.0 V	0.100+0.020	0.120+0.020	0.150+0.020	0.0050+0.0020
Continuity	1000 Ω	1 mA test current	0.002+0.010	0.008+0.020	0.010+0.020	0.0010+0.0020
Diode Test	1.0000 V [6]	1 mA test current	0.002+0.010	0.008+0.020	0.010+0.020	0.0010+0020

[1] Specifications are for 90-minute warm-up and integration setting of 100 NPLC. For <100 NPLC add the appropriate "RMS Noise Adder."

[2] Relative to calibration standards

[3] 20% overrange on all ranges except 1000 VDC, 3 A range

[4] Specifications are for 4-wire ohms function, or 2-wire ohms using Math Null. Without Math Null, add 0.2 Ω additional error in 2- wire ohms function.

[5] For each additional volt over ±500 VDC, add 0.02 mV of error.

[6] Accuracy specifications are for the voltage measured at the input terminals only. 1 mA test current is typical. Variation in the current source will create some variation in the voltage drop across a diode junction.

Figure 1. DC accuracy specifications for the Agilent 34410A DMM

Performance Versus Integration Time
60 Hz (50 Hz) power line frequency

Integration Time Number of Power Line Cycles (NPLC)	Resolution ppm Range[1]	NMR dB[2]	Readings per Second[3]	RMS Noise Adder % Range[4]		
				DCV 10, 1000 V	**DCV** 1, 100 V **Resistance** 1k, 10k Ohm	**DCV** 0.1 V **Resistance** 100 Ohm **DCI** 1 Amp
0.001[6]	30	0	50,000	0.0060	0.0100	0.1000
0.002[6]	15	0	25,000	0.0030	0.0060	0.0600
0.006	6	0	10,000	0.0012	0.0040	0.0600
0.02	3	0	3,000	0.0006	0.0030	0.0300
0.06	1.5	0	1,000	0.0003	0.0020	0.0200
0.2	0.7	0	300	0.0002	0.0015	0.0150
1	0.3	55	60(50)	0.0	0.0001	0.0010
2	0.2	110[5]	30(25)	0.0	0.0001	0.0010
10	0.1	110[5]	6(5)	0.0	0.0	0.0005
100	0.03	110[5]	0.6(0.5)	0.0	0.0	0.0

[1] Resolution is defined as the typical 10 VDC range rms noise.

[2] Normal mode rejection for power-line frequency +/- 0.1%.

[3] Maximum rate for DCV, DCI, and 2-wire resistance functions (using zero settling delay, autozero off, etc.).

[4] Autozero on for => 1 NLPC. Basic DC accuracy specifications include rms noise at 100 NPLC.
    For <100 NPLC add appropriate "RMS Noise Adder" to basic accuracy specification.

[5] For power-line frequency +/- 1% 75 dB for +/- 3% 55 dB.

[6] Only for 34411A.

Figure 2. Performance versus integration time for the Agilent 34410A DMM

for countries using 60 Hz AC line frequencies), we see that the spec is ±(0.0015% x reading + 0.0004% x range). From **Figure 2**, we also see that on the 10 V range, and for integration times of less than 100 PLC, we must add 0.0003% of range to the accuracy to account for DC noise errors. Suppose that we wanted to read a voltage that was previously calibrated, traceable to NIST standards, to be exactly 5.000000 V and we wanted to measure this on

the 10 V range at an PLC of 0.06 (1 ms). We would expect the accuracy to be ±(5.0 x 0.000015 + 10 x 0.000004 + 10 x 0.000003), or ±145 µV, and thus we would expect the DMM to display a steady reading somewhere between 4.999855 and 5.000145 volts, depending on how accurately we calibrated it. Since there is some unspecified base level of DC noise already included in

the accuracy specification and we added 30 µV of additional noise to it, we would expect to see something over ±30 µV of variation in the readings due solely to DC noise. The peak-to-peak noise is thus something over 60 µV. As this paper will show, this represents an rms noise of approximately 60 µV/6, or 10 µV. In fact, the DC noise

specification is 15 µVrms on the 10 V range, so we would infer that 5 µVrms was the unspecified DC noise that had been included in the accuracy specification.

Analog-to-digital converter performance accounts for all DC noise errors. The second column in **Figure 2** shows the DC noise specifications for this DMM. Note that it varies in relation to the number of PLCs that are chosen. For example, if NPLC=.06 (.001 second, or 1000 readings/s), the A/D converter in this DMM has an error of ±1.5 ppm rms x range. On the 10 V range, this is ±15 µVrms.

Ultimately, we want to know how many digits of resolution we can get for a given integration time. Taking the 10 V range again as an example, we would expect to see a variation from reading to reading of ±15 µV. Suppose our DMM was calibrated such that it read exactly 5.000000. We would expect readings to range from 4.999985 to 5.000015 most of the time. It is plain that the 1 µV digit is useless in this case, so we would actually expect the readings to be displayed as 4.99998 to 5.00002.

This is a 6-digit reading. Since we are on the 10 V scale and this DMM has 20% overrange capability, readings up to ±12.00000 can be taken. The most significant digit can only go from 0 to 1, so it is called a $1/2$ digit. Thus, the DMM is said to be a $6^{1}/2$-digit DMM at this reading rate, one in which the last (10 µV) digit can be expected to change from reading to reading by ±2 counts most of the time. ("Most of the time" means 68.3% of the time, which is the definition of rms.) In reality, the actual performance has been measured to be better than the spec, and we would normally see only a ±1 count change in the last digit 68.3% of the time.

The way we measure the DC noise performance is by applying a DC voltage and taking a few hundred sample readings. Since we also wish to eliminate any errors in a source, it is simplest to just short the input terminals (which gives us a 0 V input) and ignore nonlinearities that might cause some additional errors at higher input-voltage

settings. We then get a distribution in the readings that follows a statistical model. If the samples were distributed equally, we would call it a uniform distribution. A classic paper on quantization error written in 1948 by Bell Labs[1] shows that the act of quantizing an analog signal results in a uniform distribution, so if our measured readings were distributed in this way, we would say that the DMM readings were *quantization-limited*. In other words, the act of converting the analog value to digital would be the primary source of the error. If the samples were distributed in a Gaussian form (see **Figure 3**), the noise would be said to be caused by 1/f noise, thermal noise, nonlinearities and other analog errors. The manner in which we would compute the rms value is different for the two cases.

[1] W.R. Bennett, "Spectra of Quantized Signals," Bell System Technical Journal, Vol. 27, July 1948, p. 446-471.

The Bell Labs paper on quantization error shows us that the rms noise is given by:

rms noise = ideal resolution/$\sqrt{12}$

where ideal resolution = voltage span/2^n (n = number of bits in the A/D)

For example, an 8-bit A/D that can measure voltages from -1 to +1 has an ideal resolution of $(1 - (-1))/256 = 7.8$ mV, and we would expect to measure an rms noise of 2.3 mV, *not including any other analog errors due to circuitry between the input terminals and the A/D converter.*
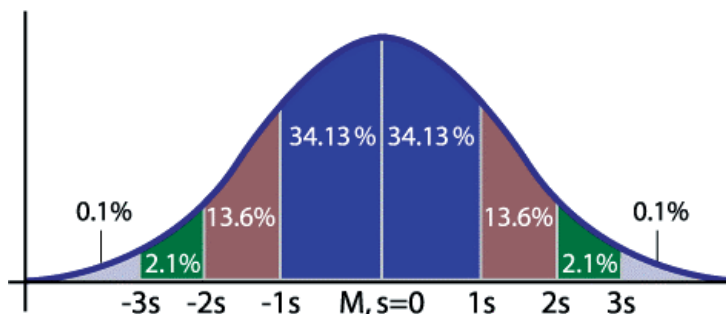

Figure 3. A Gaussian distribution

In most modern DMMs, the A/D converters have enough bits of native resolution that this quantization error is much smaller than other errors and can be ignored.

Since the other sources of error are distributed in Gaussian form, we can use the formula for standard deviation to compute the rms value.

A quick refresher on standard deviation

Suppose we are given a population x1, ..., xN of values (which are real numbers). The arithmetic mean of this population is defined as

$$\overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i = \frac{x_1 + x_2 + \cdots + x_N}{N}$$

and the standard deviation of this population is defined as

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})^2}$$

Given only a sample of values x1,...,xN from some larger population, many authors define the sample standard deviation by

$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})^2}.$$

Since we are using a 0 V input signal, the mean is defined to be 0, so we can use the calculated sample standard deviation as the value of rms error.
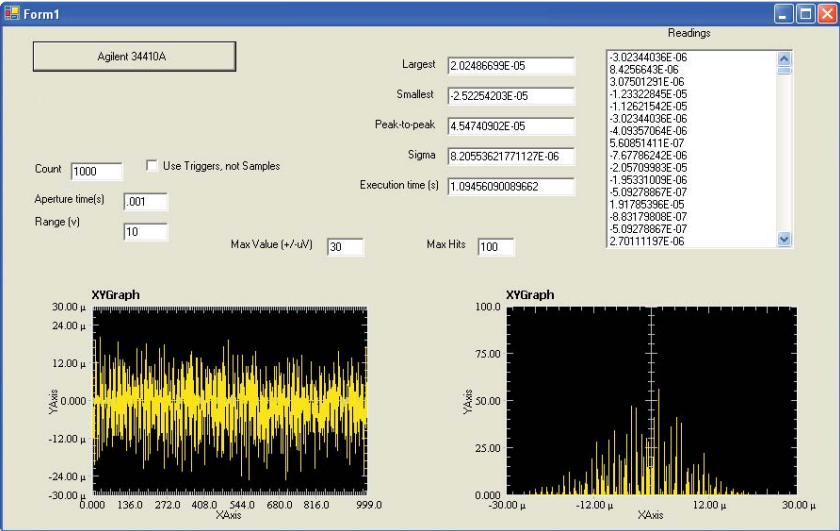
$$rms = \sqrt{\overline{x}^2 + s^2}$$

$$\overline{x} = 0, \text{ so}$$

$$rms = \sqrt{s^2} = s$$

A Visual Studio®.NET program was written to control a 34410A DMM using Agilent's T&M Toolkit software to make the graphing job and the instrument control very easy. The inputs were shorted and 1000 samples taken on the 10 V range with an aperture time of 1 ms. The data were plotted as raw data and as a histogram, as shown in **Figure 4**. Although quantization "buckets" are clearly visible in the graph on the right, the overall pattern is that of a Gaussian distribution. This DMM is therefore *not* quantization-limited.

Figure 4. DC noise for the Agilent 34410A DMM

In this case, 8.2 µVrms, or 45.5 µVp-p (±22.8 µV) of DC noise was measured. (Remember that the spec was 15 µVrms, so it is well within this limit.) Thus, there is a 68.3% probability that the measured value will be within ±8.2 µV of the actual value. A peak-to-peak value can be called three things that all have the same meaning: ±3 sigma, six-sigma, or 99.7% probability. Thus there's a 99.7% probability that the measured value will be within ±22.8 µV of the measured value. Since these measurements were taken on the 10 V range, the reading would best be displayed as 0.00000, and we might expect to see it change from -0.00001 to +0.00001 once in a while and go as high as ±0.00002 on a less-frequent basis.

An effective number of digits (ENOD) can be calculated as follows:

ENOD = $\log_{10}$ (positiveSpan/rmsNoise) for a 68.3% probability (±1 sigma)

or

ENOD = $\log_{10}$ (totalSpan/ppNoise) for a 99.7% probability (±3 sigma)

Companies do not usually publish the internal span of their A/D converters, so it may not be possible to derive this number when you compare specs from one company to another. In the case of the 34410A, the internal range is ±14 V on the 10 V range, even though the usable range is artificially limited to ±12 V. So, plugging in,

ENOD = $\log_{10}$ (14/.0000082) = 6.2 digits (68.3% probability)

or

ENOD = $\log_{10}$ (28/.0000455) = 5.8 digits (99.7% probability)

Since the spec is based on rms noise, not peak-to-peak noise, we would call this a $6\frac{1}{2}$-digit DMM on the 10 V range at 1000 readings/s. The same method can be used to calculate the ENOD for other ranges and speeds.

One can also use signal-to-noise + distortion ratio (SINAD) to calculate ENOD. There is a well-known formula relating signal, noise, harmonic distortion and effective number of bits in the A/D:

N (effective) = (S/(N + D) – 1.8)/6.02

Where S/(N+D) is the ratio of the signal power of a full-scale input to the total power of noise plus distortion, expressed in dB[2]. This formula reinforces the notion that noise errors come from many sources, not merely quantization error. A full discussion of this method of measurement is beyond the scope of this paper.

[2] David A. Czenkusch, "High Resolution Digitizing Techniques with an Integrating Digital Multimeter", Hewlett Packard Journal, April, 1989, Vol. 40, Nr. 2, pp.39-49.

Conclusion
Establishing the number of usable digits for a DMM depends upon its calibrated accuracy and the performance of its A/D converter. In most modern DMMs, the quantization error due to analog-to-digital conversion is far less than that of other analog errors and can be ignored. The effective number of digits is based on rms noise performance, which is a Gaussian distribution of noise that is generated inside the DMM by thermal noise, 1/f noise, harmonic distortion and other analog errors.

Related Agilent literature

Product Overview
5989-3738EN
Agilent 34410A and 34411A Multimeters
http://cp.literature.agilent.com/ litweb/pdf/5989-3738EN.pdf

Application Note
5989-4279ENUS
8 Hints for Making Better Digital Multimeter Measurements
http://cp.literature.agilent.com/ litweb/pdf/5989-4279ENUS.pdf

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